Active Low-Pass Filter

Abstract: Active Filters, Flat and Bump

The following chapters are used as class notes in an analog-circuit course in a two-year electrical-engineering-technology program at a community college. Chapter 1 presents the active flat (Butterworth) filter. None of that material is unique – it is used as a lead-in for Chapter 2, "Active Low-Pass Bump Filter".

Chapter 2 presents some equations, ***believed to be unique***, for a "bump" (Chebyshev) filter, the counterpart of the flat filter. When the "bump" magnitude and cut-off frequency are specified, equations are given for the necessary resistor and capacitor values. Alternatively, if the R & C vales are given, straight-forward equations are presented to calculate the "bump" magnitude and cut-off frequency. Finally, examples are given for multi-stage filters that are combinations of "bump" and flat filters.

Keywords: active filters, active low-pass filters, Sallen-Key filters, Chebyshev filters, low-pass filters.

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Chapter 1 Flat (Butterworth) Active Filter

Texas Instrument's ***SLOA024B***, "Analysis of Sallen-Key Architecture" by James Karki shows the circuit below. It is a 2nd-order active low-pass filter. "Figure 2 shows a unity gain amplifier used in this manner. Capacitor C2 provides a positive feedback path. In 1955, R. P. Sallen and E. L. Key described these filter circuits, and hence they are generally known as Sallen-Key filters."

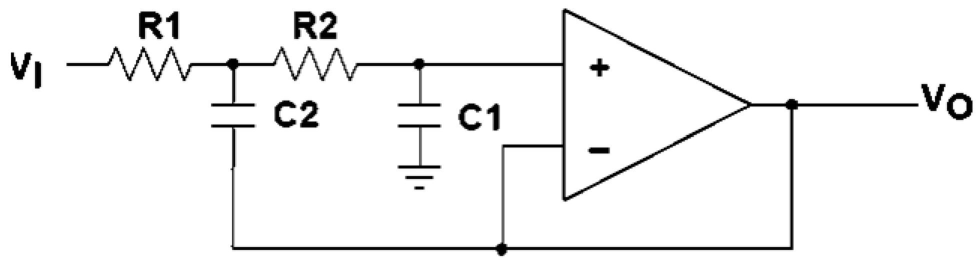

Figure 2. Unity Gain Sallen-Key Low-Pass Filter

"The operation can be described qualitatively:

- At low frequencies, where C1 and C2 appear as open circuits, the signal is simply buffered to the output.
- At high frequencies, where C1 and C2 appear as short circuits, the signal is shunted to ground at the amplifier's input, the amplifier amplifies this input to its output, and the signal does not appear at Vo.
- Near the cut-off frequency, where the impedance of C1 and C2 is on the same order as R1 and R2, positive feedback through C2 provides Q enhancement of the signal." In fact, this chapter shows that if C2 = 2C1, the frequency response is flat in the passband. Chapter 2 shows that if C2 > 2C1, the frequency response is not flat, rather it has a "bump" in the passband."

The following transfer function is given:

$$\frac{V_o}{V_i} = \frac{1}{s^2(R_1R_2C_1C_2) + sC_1(R_1 + R_2) + 1}$$

Define the denominator of this equation as:

$$D(s) = s^2(R_1R_2C_1C_2) + sC_1(R_1 + R_2) + 1.$$

To simplify the design, set $R_1 = R_2 = R$.

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$D(s) = s^2(R^2C_1C_2) + 2sRC_1 + 1.$

Introduce two new parameters, Q and ω_p , defined below:

$$\omega_p = \frac{1}{R\sqrt{C_1C_2}}; \quad Q = \sqrt{\frac{C_2}{4C_1}}; \quad Q^2 = \frac{C_2}{4C_1}; \quad \frac{1}{Q^2} = \frac{4C_1}{C_2}.$$

$$D(s) = \frac{s^2}{\omega_p^2} + \frac{s}{Q\omega_p} + 1.$$

Let $s = j\omega$:

$$D(j\omega) = -\frac{\omega^2}{\omega_p^2} + \frac{j\omega}{Q\omega_p} + 1.$$

To get the magnitude of $D(j\omega)$, use the Pythagorean theorem:

$$[D(\omega)]^2 = (1 - \frac{\omega^2}{\omega_p^2})^2 + \frac{\omega^2}{Q^2\omega_p^2} = 1 - \frac{\omega^2}{\omega_p^2}\left(2 - \frac{1}{Q^2}\right) + \frac{\omega^4}{\omega_p^4}.$$

$\omega = 2\pi F$; $\omega_p = 2\pi F_p$,

where $\boldsymbol{F_p = \frac{1}{2\pi R\sqrt{C_1C_2}}}$.

$$[D(F)]^2 = 1 - \frac{F^2}{F_p^2}\left(2 - \frac{1}{Q^2}\right) + \frac{F^4}{F_p^4}.$$

$$\boldsymbol{[D(F)]^2 = 1 - \frac{F^2}{F_p^2}\left(2 - \frac{4C_1}{C_2}\right) + \frac{F^4}{F_p^4}}.$$

The magnitude of the gain $= \frac{V_o}{V_i} = \frac{1}{\sqrt{[D(F)]^2}} = \frac{1}{\sqrt{1 - \frac{F^2}{F_p^2}\left(2 - \frac{4C_1}{C_2}\right) + \frac{F^4}{F_p^4}}}$

The flat (Butterworth) (maximally flat) low-pass filter has a characteristic shown in the two plots on the next page. In this example, the cut-off frequency = F_c = 1 kHz. The pass-band gain is 0dB until it rolls off to -3dB at F_c. Then the attenuation band gain falls off at -24dB/octave = -40dB/decade. Note that the gain at 4 kHz (2 octaves beyond F_c) is -24dB.

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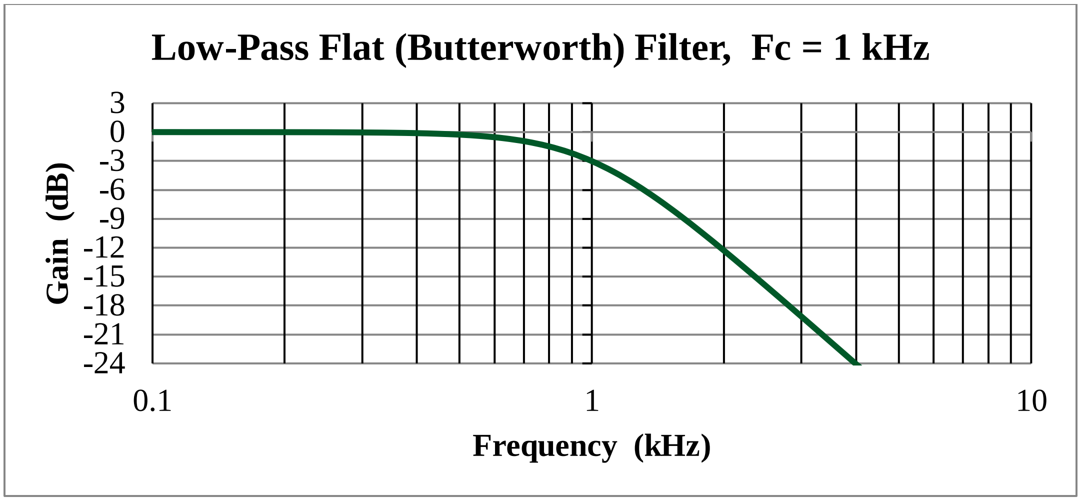

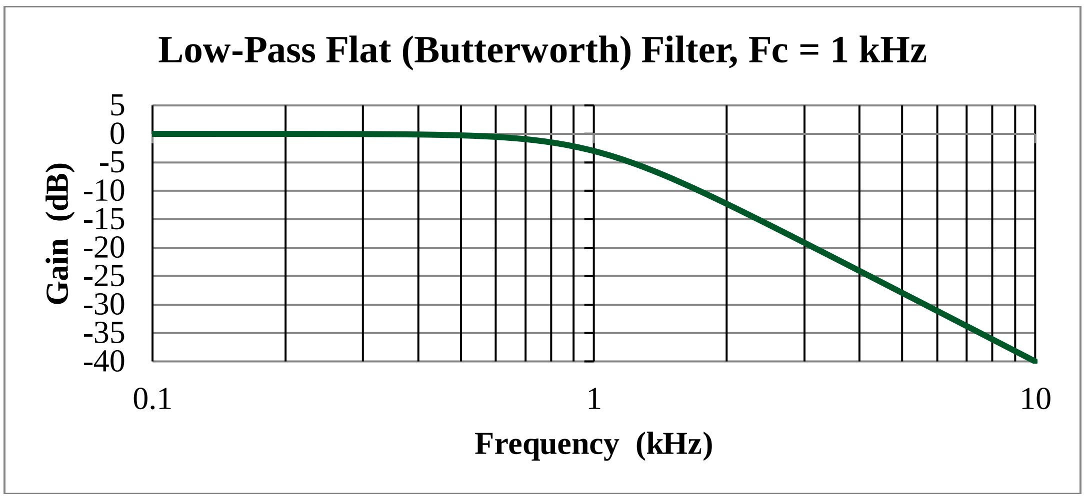

Note that the gain at 10 kHz (1decade beyond F_c) is -40dB.

The equation to produce the above plot is:

$$20\log\frac{V_o}{V_i} = 20\log\frac{1}{\sqrt{1\ +\ (F/F_C)^4}} = 20\log\frac{1}{\sqrt{[D(F)]^2}}.$$

Thus

$$[D(F)]^2 = 1 + \frac{F^4}{F_C^4}\ . \quad \text{(Flat Filter Equation)}$$

On page 3, however, there is the following equation:

$$[D(F)]^2 = 1 - \frac{F^2}{F_p^2}\left(2 - \frac{4C_1}{C_2}\right) + \frac{F^4}{F_p^4}. \quad \text{(Sallen-Key Equation)}$$

$$[D(F)]^2 = 1 + \frac{F^4}{F_C^4} = 1 - \frac{F^2}{F_p^2}\left(2 - \frac{4C_1}{C_2}\right) + \frac{F^4}{F_p^4}.$$

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Equate coefficients of F^4:

$$\frac{1}{F_c^4} = \frac{1}{F_p^4}; \quad or\ F_c = F_p.$$

Equate coefficients of F^2:

$$0 = 2 - \frac{4C_1}{C_2}. \quad \text{Or } 2 = \frac{4C_1}{C_2}.$$

$$C_2 = 2\ C_1.$$

$$F_c = \frac{1}{2\pi R\sqrt{C_1 C_2}} = \frac{1}{2\pi R\sqrt{2C_1^2}} = \frac{1}{2\sqrt{2}\pi R C_1}.$$

Design Example: Design a flat filter for $F_c = 1\ kHz$.

Use the two equations:

$$C_2 = 2\ C_1.$$

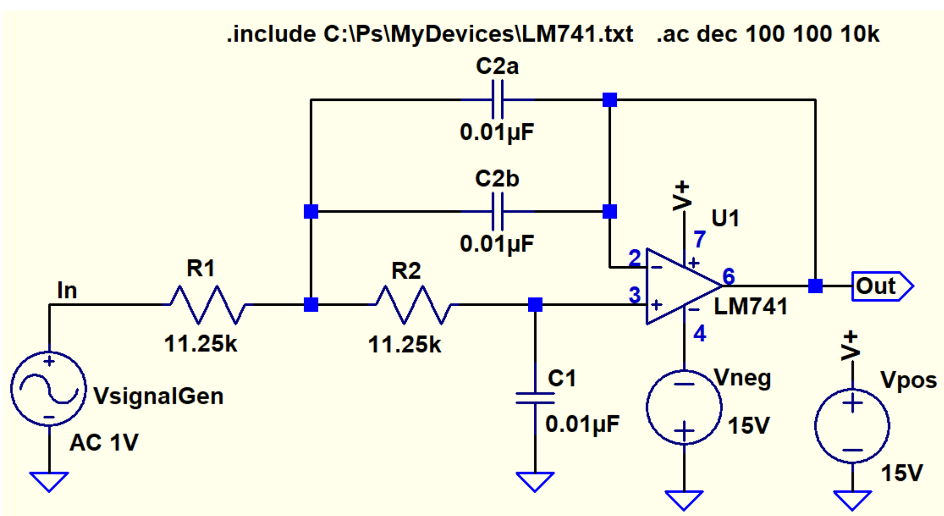
$$F_c = \frac{1}{2\sqrt{2}\pi R C_1}.$$

Select $C_1 = 0.01\mu F$.

Calculate $C_2 = 2\ C_1 = 0.02\mu F$. Use two $0.01\mu F$ caps in parallel for C_2.

Solve for R:

$$R = \frac{1}{2\sqrt{2}\pi F_c C_1} = \frac{1}{2\sqrt{2}\pi(1E03)(0.01E-06)} = 11.25k\Omega.$$



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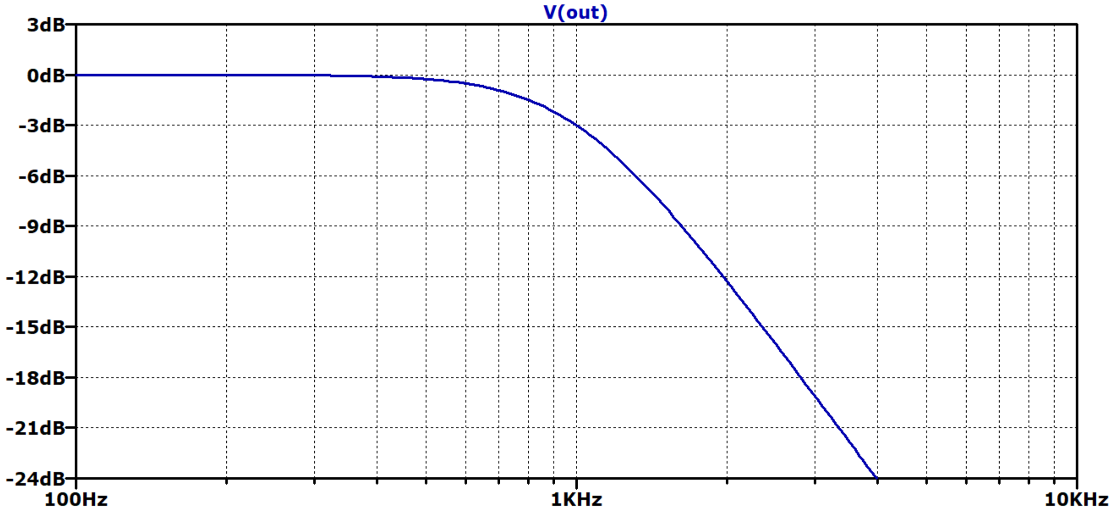


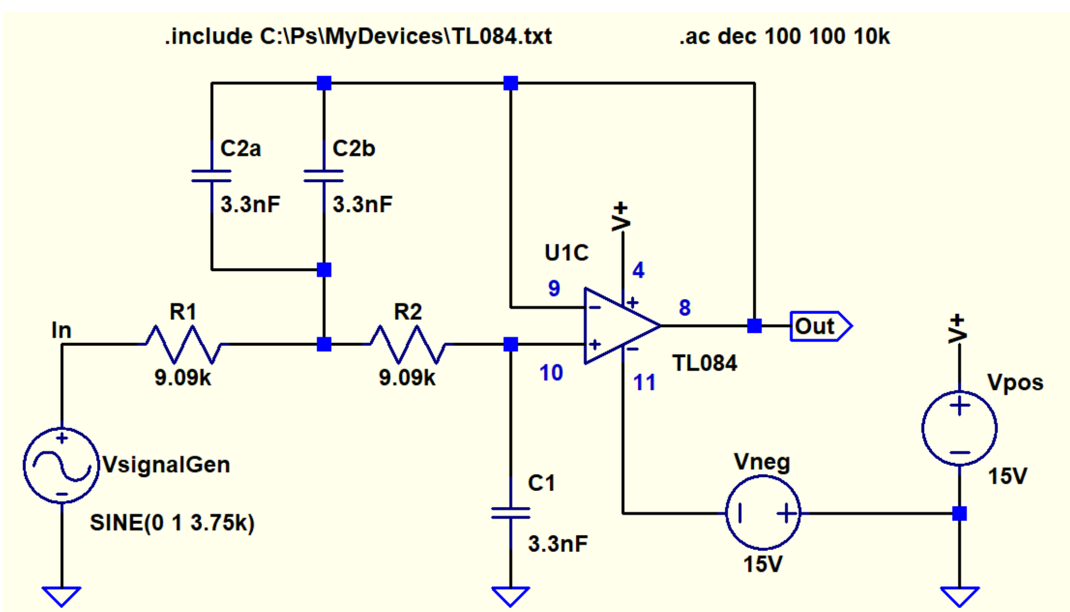
LTspice Circuit and Plot.

$$20\log\frac{V_o}{V_i} = 20\log\frac{1}{\sqrt{1 \;+\; (F/F_c)^4}}$$

Using the algebra of logs:

$$20\log\frac{V_o}{V_i} = -10\log\,[1 \;+\; (F/F_c)^4]$$

Example 2: Analysis Problem. A low-pass flat (Butterworth) filter is shown below. For the component values shown, calculate F_c, using the formula on page 5. Does your answer match the answer shown on the next page?



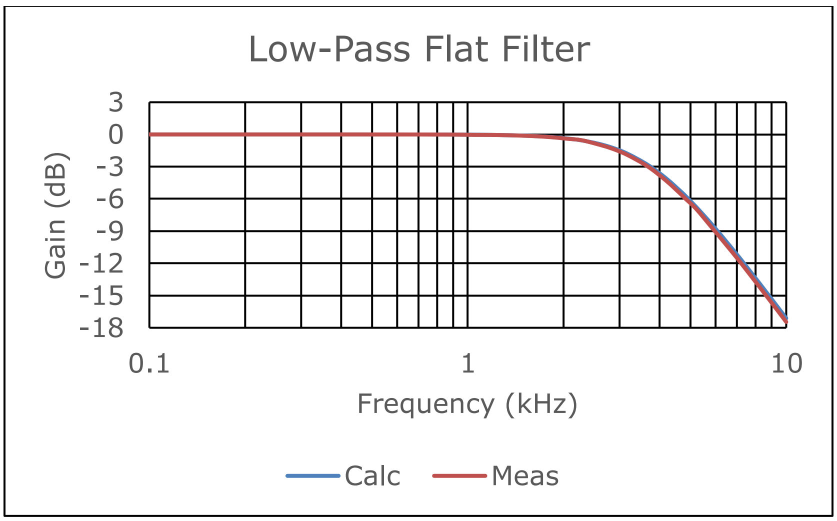
Answer:

$F_c = \frac{1}{2\sqrt{2}\pi RC_1}$ =3.75 kHz.

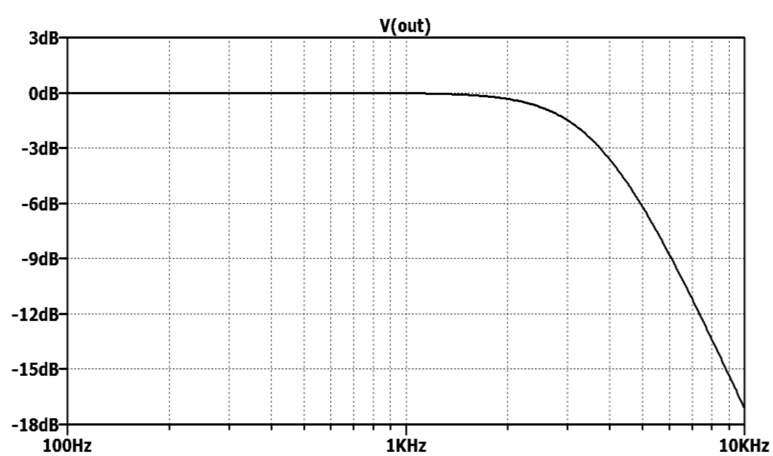
- Calculate the gain, in dB at 6 kHz.

$$20\,log\,\frac{V_o}{V_i} = -10\,log\,\left[1 \quad + \quad (F/F_c)^4\right].$$

$$20\,log\,\frac{V_o}{V_i} = -10\,log\,\left[1 \quad + \quad (6/3.75)^4\right] = \mathbf{-8.77\ dB}.$$



Excel Plot from the gain formula: $20\log\frac{V_o}{V_i} = -10\log\left[1 \quad + \quad (F/F_c)^4\right]$



LTspice Plot from an AC analysis of the circuit on the previous page.

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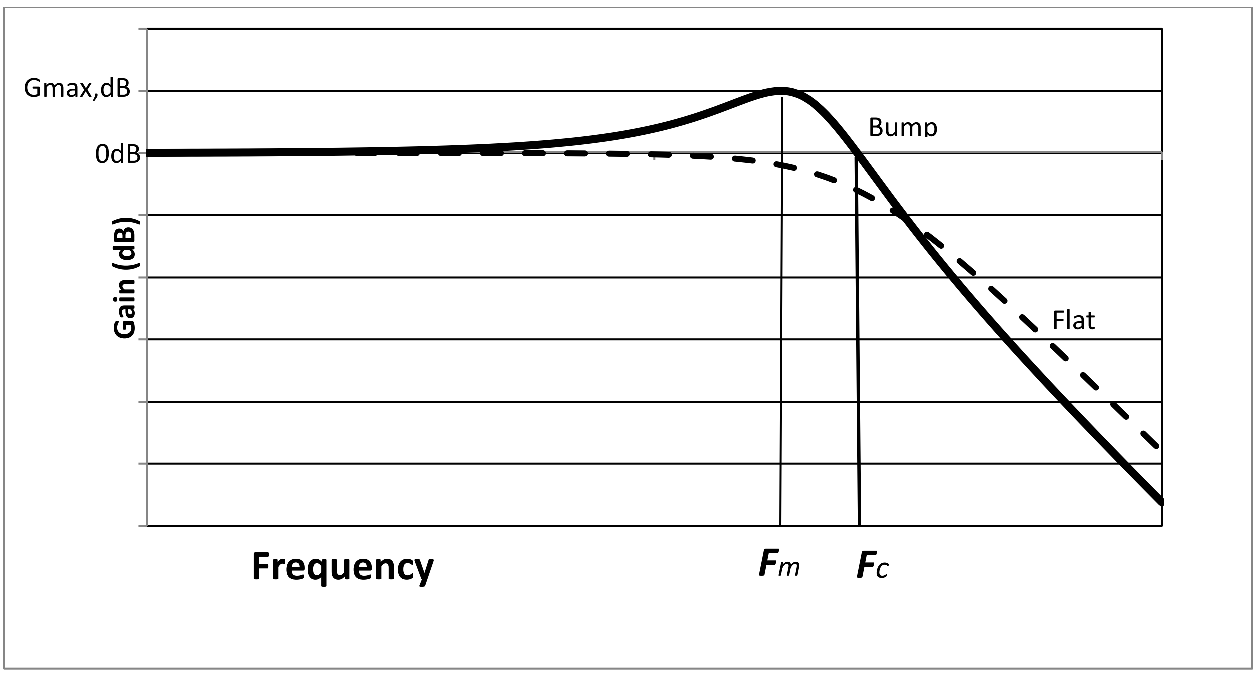
Chapter 2: Active Low-Pass Bump Filter

Abstract: This chapter derives a set of successive equations for a “bump” low-pass Sallen-Key active filter. The equations start as a design where the magnitude of the “bump”, $G_{max,dB}$, and the cutoff frequency, F_c, are specified. Equations are then derived to solve for the required R & C values. Later, the reverse process is shown where the R & C values are given, and equations are derived to solve for the values of $G_{max,dB}$ and F_c.

Introduction:
Sometimes there is an advantage to using a low-pass (LP) filter with a “bump” in its “Bode Plot”. The 2nd order flat (Butterworth) filter’s gain is simply flat in its pass band, and then declines at 40dB/decade in its attenuation band. The 2nd order “bump” filter’s gain declines at more than 40dB/decade, but at the expense of being non-flat, or having a “bump” in the pass band. See diagram below. The larger the “bump”, the more the gain declines in the attenuation band.



When several filters are connected together in tandem, there is often an advantage to using a combination of “bump” and flat filters to get the desired overall frequency response.

Comparison of the "Bump" Filter to the Chebyshev Filter:

Excerpted from Texas Instrument's SBFA001C "FilterPro™ User's Guide":

Chebyshev (Equal Ripple Magnitude)

Note: Mr. Chebyshev's name is also transliterated *Tschebychev*, *Tschebyscheff*, or *Tchevysheff*. "This filter response has steeper attenuation above the cutoff frequency than Butterworth. This advantage comes at the penalty of amplitude variation (ripple) in the passband. Unlike Butterworth and Bessel responses, which have 3dB attenuation at the cutoff frequency, the Chebyshev cutoff frequency is defined as the frequency at which the response falls below the ripple band. For even-order filters, all ripple is above the 0dB-gain dc response, so cutoff is at 0dB (as Figure 5 shows). "

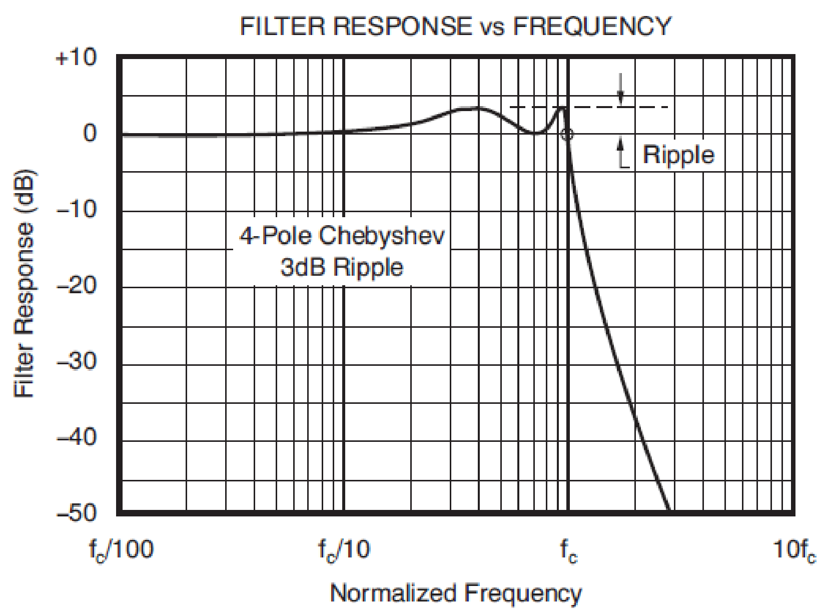


Figure 5. Response vs Frequency of Even-Order (Four-Pole), 3dB Ripple Chebyshev Filter Showing Cutoff at 0dB

1. The gain is $0dB$ at F_c, *the cut-off frequency*, for both the "bump" and Chebyshev filters. See above diagrams.
2. $G_{max,dB}$, the "bump," is called "ripple" for the Chebyshev filter.
3. The "bump" filter has a straight-forward set of design equations presented in this chapter. The set of equations are easy to use, and ***believed to be unique***.

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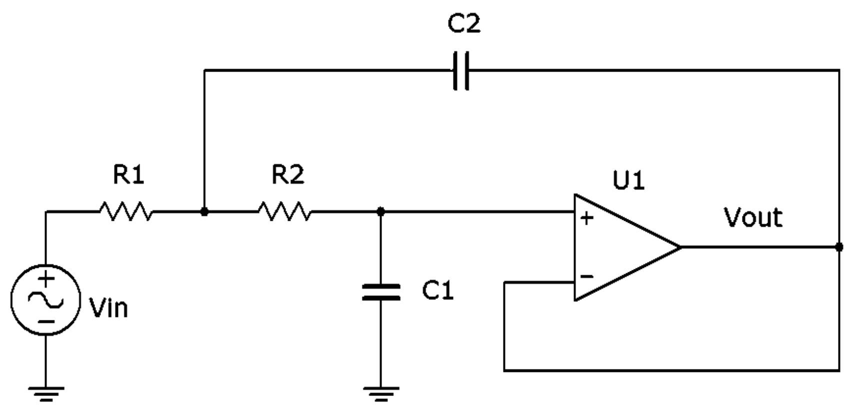
The "bump" filter has the Sallen-Key-filter topology.



Gain Equations

$$G_{dB} = 20 \log \frac{V_{out}}{V_{in}} = 20 \log G$$

The bump filter's gain equation, ***believed to be unique*** is:

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{[D(F)]^2}} = \frac{1}{\sqrt{1 - x\ (F/F_c)^2 + x\ (F/F_c)^4}}.$$

Where x is a parameter that is a function of the "bump", $G_{max,dB}$.

If $F = F_c$,

$$G_{dB} = 20\ log\ \frac{1}{\sqrt{1 - x + x}} = 20\ log\ \frac{1}{1} = 0dB.$$

Thus F_c is the frequency at which the gain is 0dB, not -3dB.

Define: $[D(F)]^2 = 1 - x\ (F/F_c)^2 + x\ (F/F_c)^4$.

Define: $[D(F)]^2_{min}$= the minimum value of $[D(F)]^2$.

$$G_{max,dB} = 20 \log G_{max} = 20 \log \frac{1}{\sqrt{[D(F)]^2_{min}}}.$$

That is, gain is at its max value when the denominator is at its minimum value.

When $F = F_m$, $[D(F)]^2 = [D(F)]^2_{min}$. See diagram on page 8.

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Proof that $F_m = F_c/\sqrt{2}$.

$[D(F)]^2 = 1 - x\ (F/F_c)^2 + x\ (F/F_c)^4.$

To get the minimum value of $[D(F)]^2$ with respect to F,

- Derive the equation for the derivative of $[D(F)]^2$ with respect to F.
- Set the derivative equal to zero.

$[D(F)]^2 = 1 - x\ (F/F_c)^2 + x\ (F/F_c)^4.$

To simplify use: $y = (F/F_c)^2$.

$[D(F)]^2 = 1 - x\,y + x\,y^2.$

$\frac{d[D(F)]^2}{dy} = -x + 2xy.$ The derivative of $[D(F)]^2$ with respect to y.

$-x + 2xy = 0.$ The derivative is set equal to zero.

$-1 + 2y = 0.$ Cancel x.

$y = 1/2.$

$y = (F/F_c)^2 = 1/2.$

$F/F_c = 1/\sqrt{2}.$

$F = F_c/\sqrt{2}.$

$F_m = F_c/\sqrt{2}.$

Repeat of the previous equation:

$[D(F)]^2 = 1 - x\ (F/F_c)^2 + x\ (F/F_c)^4.$

Substitute $F_m = F_c/\sqrt{2}$, to get $[D(F)]^2_{min}$:

$[D(F)]^2_{min} = 1 - x\ (F_m/F_c)^2 + x\ (F_m/F_c)^4.$

$[D(F)]^2_{min} = 1 - x\ (F_c/\sqrt{2}F_c)^2 + x\ (F_c/\sqrt{2}F_c)^4.$

$[D(F)]^2_{min} = 1 - x\ (1/\sqrt{2})^2 + x\ (1/\sqrt{2})^4.$

$[D(F)]^2_{min} = 1 - x/2 + x/4.$

$[D(F)]^2_{min} = 1 - x/4.$

$\boldsymbol{x = 4(1 - [D(F)]^2_{min}) = 4(1 - 1/G^2_{max})}$.

$$G_{max,dB} = 20\log\frac{1}{\sqrt{[D(F)]^2_{min}}} = 20\log\frac{1}{\sqrt{1 - x/4}} = 10\log\frac{1}{1 - x/4}.$$

Using the algebra of logs:

$$G_{max,dB} = -10\log\ (1 - x/4).$$

$$G_{dB} = 20\log G = 20\log\frac{1}{\sqrt{1 - x\,(F/F_c)^2 + x\,(F/F_c)^4}} = \frac{1}{\sqrt{[D(F)]^2}}.$$

Using the algebra of logs:

$$G_{dB} = -10\log\ [1 - x\ (F/F_c)^2 + x\ (F/F_c)^4].$$

Thus :

$[D(F)]^2 = 1 - x\ (F/F_c)^2 + x\ (F/F_c)^4$. (Bump Filter Equation)

In Chapter 1, however, there is the following equation:

$[D(F)]^2 = 1 - \frac{F^2}{F_p^2}\left(2 - \frac{4C_1}{C_2}\right) + \frac{F^4}{F_p^4}$ (Sallen-Key Equation)

$$[D(F)]^2 = 1 - x\ (F/F_c)^2 + x\ (F/F_c)^4 = 1 - \frac{F^2}{F_p^2}\left(2 - \frac{4C_1}{C_2}\right) + \frac{F^4}{F_p^4}$$

Equate coefficients of F^4:

$\frac{x}{F_c^4} = \frac{1}{F_p^4}$. or $F_p = F_c/x^{0.25}$ by taking the 4th root of both sides.

Or $F_p^2 = F_c^2/\sqrt{x}$ by taking the square root of both sides.

Equate coefficients of F^2:

$$\frac{x}{F_c^2} = \frac{1}{F_p^2}(2 - \frac{4C_1}{C_2}).$$

$\frac{x}{F_c^2} = \frac{\sqrt{x}}{F_c^2}(2 - \frac{4C_1}{C_2})$ by substituting $F_p^2 = F_c^2/\sqrt{x}$.

$$\sqrt{x} = 2 - \frac{4C_1}{C_2}.$$

$\frac{4C_1}{C_2} = 2 - \sqrt{x}$.

$$\boldsymbol{C_1 = \frac{C_2\ (2-\sqrt{x})}{4} = 0.25C_2\ \left(2 - \sqrt{x}\right).}$$

Design Equations

In a design, the values of $G_{max,dB}$ and F_c are specified. That is, the value of the "bump" and the cut-off frequency are specified.

1. Calculate $F_m = F_c/\sqrt{2}$.
2. Calculate $G_{max} = 10^{G_{max,dB}/20}$.
3. Calculate $x = 4(1 - 1/G_{max}^2), \quad x < 4$.
4. Select a standard value for C_2 .
5. Calculate $F_p = F_c/x^{0.25}$.
6. Calculate $C_1 = \frac{C_2\ (2-\sqrt{x})}{4} = 0.25C_2\ \left(2 - \sqrt{x}\right), \quad x < 4$.
7. Calculate $R = \frac{1}{2\pi F_p\sqrt{C_1 C_2}}$. (From Chapter 1)

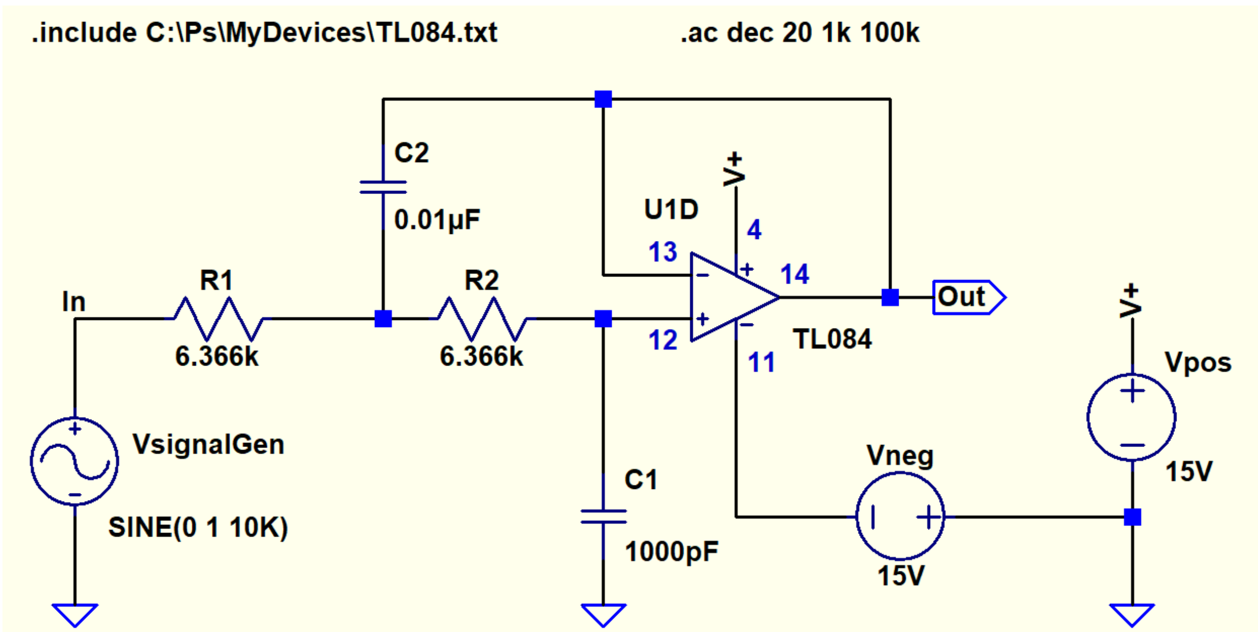
Design Example: Design a LP filter for $\boldsymbol{F_c = 10kHz}$ *and* $\boldsymbol{G_{max,dB}}$ ***= 4.437dB.***

Select $\boldsymbol{C_2}$ ***= 0.01μF = 10nF.***

1. $F_m = F_c/\sqrt{2} = 10kHz/\sqrt{2} = 7.07\ kHz$.
2. $G_{max} = 10^{\frac{G_{max,dB}}{20}} = 10^{\frac{4.437}{20}} = 1.6667$.
3. $1/G_{max}^2 = 1/1.6667^2 = 0.3600$.
4. $x = 4(1 - 1/G_{max}^2) = 4(1 - 0.3600) = 2.560$.
5. $F_p = \frac{F_c}{x^{0.25}} = \frac{10kHz}{2.560^{0.25}} = 7.906\ kHz$.

6. $C_1 \;=\; 0.25C_2\,\left(2-\sqrt{x}\right) = 0.25*0.01\mu F\left(2-\sqrt{2.560}\right) =$
   $0.001\mu F \;=1000pF \;=1nF.$

7. $R \;= \frac{1}{2\pi F_p\sqrt{C_1C_2}} = \frac{1}{2\pi*7.906E03\sqrt{0.001E-06*0.01E-06}} =\; 6.366\text{k}\Omega.$



LTspice Circuit

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Excel Calculations for

$$G_{dB} = 20log\,(\frac{V_{out}}{V_{in}}) = -10\,log\,\left[1 - x\,(F/F_c)^2 + x\,(F/F_c)^4\right]$$

Freq.	Gain		Steps/dec=	20	
kHz	dB				
1	0.11		Gmax (dB)	4.437	Given
1.12	0.14		Gmax	1.6667	Calc.
1.26	0.18		1/Gmax^2	0.3600	Calc.
1.41	0.22		x	2.560	Calc.
1.58	0.28		Fc (kHz)	10	Given
1.78	0.35		Fm (kHz)	7.07	Calc.
2.00	0.45		Fp (kHz)	7.906	Calc.
2.24	0.56		C2 (μF)	0.01	Select
2.51	0.71		C1 (μF)	0.001000	Calc.
2.82	0.90		C1 (pF)	1000	Calc.
3.16	1.14		R (kΩ)	6.366	Calc.
3.55	1.44				
3.98	1.81				
4.47	2.28				
5.01	2.85				
5.62	3.50				
6.31	4.13				
7.08	4.44				
7.94	3.94				
8.91	2.35				
10.00	0.00				
11.22	-2.64				
11.40	-3.00				
12.59	-5.28				
14.13	-7.84				
15.85	-10.30				
17.78	-12.67				
19.95	-14.97				
22.39	-17.20				
25.12	-19.38				
28.18	-21.53				
31.62	-23.64				
35.48	-25.73				
39.81	-27.81				
44.67	-29.86				
50.12	-31.91				
56.23	-33.94				
63.10	-35.97				
70.79	-38.00				
79.43	-40.01				
89.13	-42.03				
100.00	-44.04				

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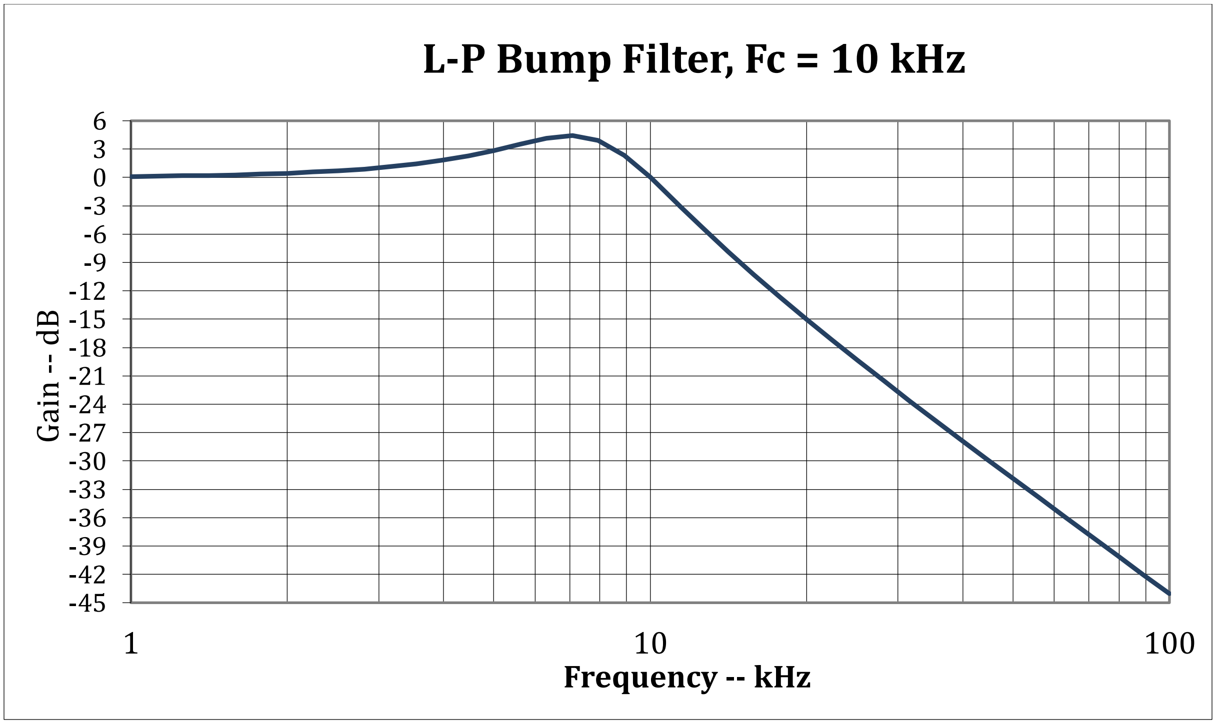
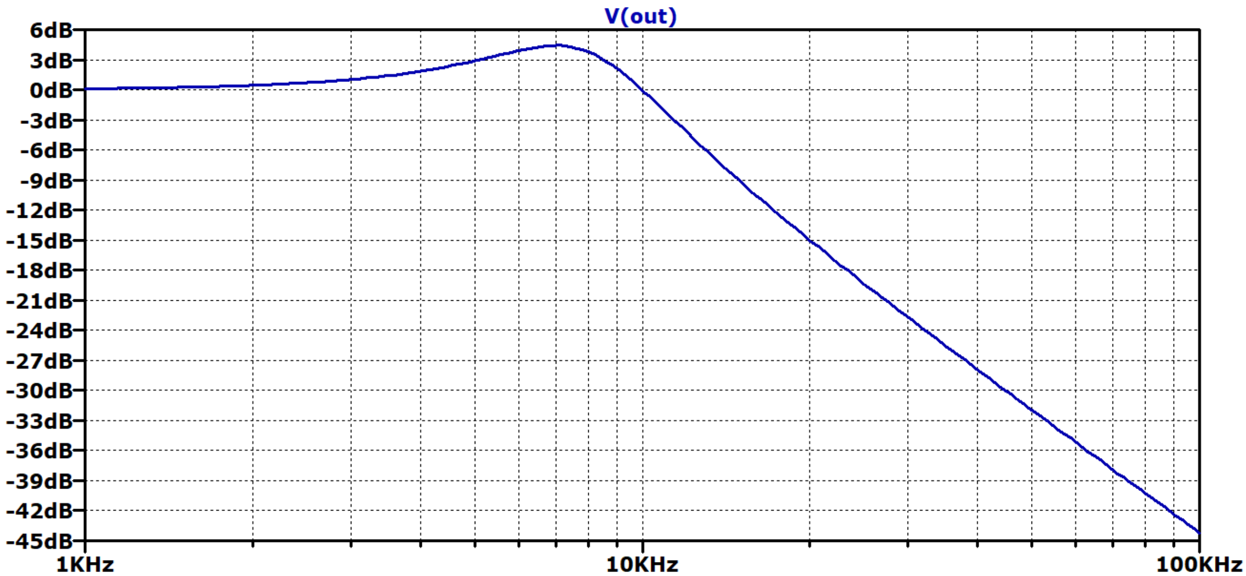


- Excel plotted the upper graph using the equation

$$G_{dB} = 20log\,(\frac{V_{out}}{V_{in}}) = -10\,log\ [1\ - x\ \ (F/F_c)^2\ +\ x\ \ (F/F_c)^4].$$

- LTspice plotted the lower graph using the R & C values in the circuit on pg7.
- The two curves look identical, proving that the design equations are correct.



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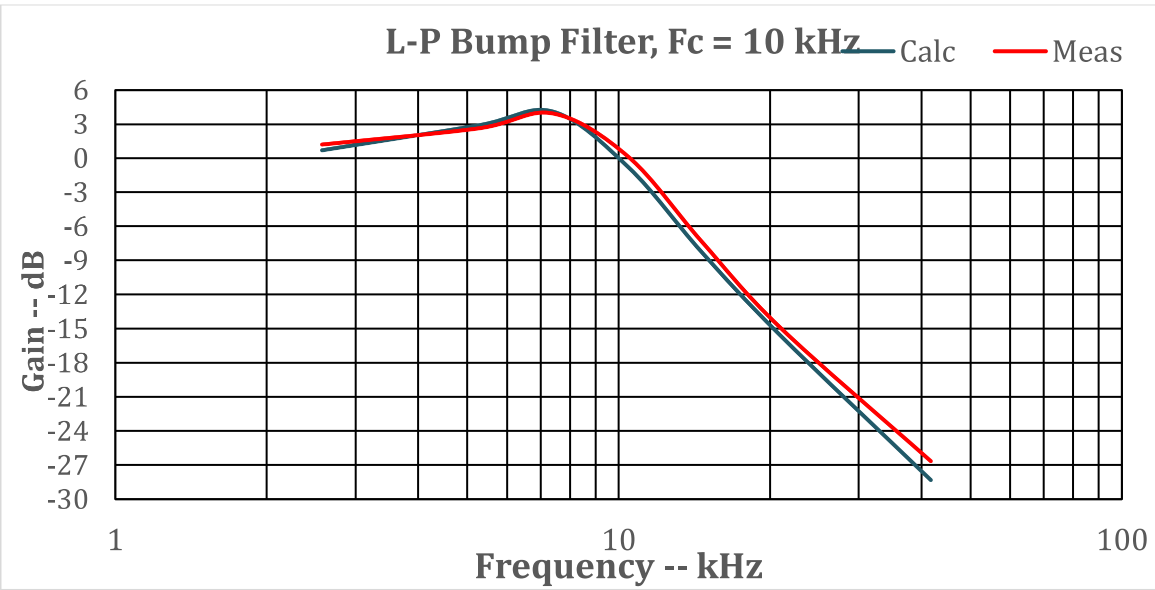
Measured Results:

			Freq	dB	dB	Vin	Vout
	Meas		kHz	Calc	Meas	mV, rms	mV, rms
R1 (kΩ)	6.29		2.5750	0.71	1.21	415	477
R2 (kΩ)	6.26		5.2480	2.89	2.62	417	564
C1 (nF)	1.034		7.4122	4.10	3.94	411	647
C2 (nF)	9.588		10.484	-0.84	0.09	400	404
	Calc		14.605	-8.23	-7.30	403	174
x	2.461		20.954	-15.58	-14.90	403	72.5
Gmax (dB)	4.147		41.715	-28.31	-26.65	400	18.6
Fp (kHz)	8.055						
Fc (kHz)	10.089						



Capacitor selection:

Some values of $G_{max,dB}$ could result in non-standard capacitor values.

From Page 12: $\sqrt{x} = 2 - \frac{4C_1}{C_2}$.

Square both sides to solve for x:

$$x = [2 - 4\frac{C_1}{C_2}]^2, \quad C_2 > 2C_1.$$

$$1/G_{max}^2 = 1 - {}^{x}/_{4}, \quad x < 4.$$

$$G_{max,dB} = 20LOG(G_{max}) = -10LOG(\,1/G_{max}^2) = -10LOG(1 - {}^{x}/_{4}).$$

Active Low-Pass Filter

These equations are used to generate this **Capacitor Standard Value Table**.

C1	C2	n =			Gmax
pF	µF	C2/C1	Q	x	(dB)
4700	0.010	2.128	0.7293	0.0144	0.016
10000	0.022	2.200	0.7416	0.0331	0.036
6800	0.015	2.206	0.7426	0.0348	0.038
4300	0.010	2.326	0.7625	0.0784	0.086
9100	0.022	2.418	0.7774	0.1193	0.132
6200	0.015	2.419	0.7777	0.1202	0.132
3900	0.010	2.564	0.8006	0.1936	0.215
5600	0.015	2.679	0.8183	0.2567	0.288
8200	0.022	2.683	0.8190	0.2592	0.291
3600	0.010	2.778	0.8333	0.3136	0.355
7500	0.022	2.933	0.8563	0.4050	0.464
5100	0.015	2.941	0.8575	0.4096	0.469
3300	0.010	3.030	0.8704	0.4624	0.534
4700	0.015	3.191	0.8932	0.5575	0.652
6800	0.022	3.235	0.8993	0.5831	0.684
3000	0.010	3.333	0.9129	0.6400	0.757
4300	0.015	3.488	0.9339	0.7282	0.873
6200	0.022	3.548	0.9419	0.7617	0.917
2700	0.010	3.704	0.9623	0.8464	1.033
3900	0.015	3.846	0.9806	0.9216	1.137
5600	0.022	3.929	0.9910	0.9640	1.198
2400	0.010	4.167	1.0206	1.0816	1.369
3600	0.015	4.167	1.0206	1.0816	1.369
5100	0.022	4.314	1.0385	1.1507	1.473
2200	**0.010**	**4.545**	**1.0660**	**1.2544**	**1.634**
3300	0.015	4.545	1.0660	1.2544	1.634
4700	0.022	4.681	1.0818	1.3121	1.726
2000	0.010	5.000	1.1180	1.4400	1.938
3000	0.015	5.000	1.1180	1.4400	1.938
4300	0.022	5.116	1.1310	1.4840	2.013
1800	0.010	5.556	1.1785	1.6384	2.289
2700	0.015	5.556	1.1785	1.6384	2.289
3900	0.022	5.641	1.1875	1.6664	2.340
3600	0.022	6.111	1.2360	1.8102	2.617
2400	0.015	6.250	1.2500	1.8496	2.695
1500	0.010	6.667	1.2910	1.9600	2.924

Active Low-Pass Filter

C1	C2	n =			Gmax
pF	µF	C2/C1	Q	x	(dB)
3300	0.022	6.667	1.2910	1.9600	2.924
10000	0.068	6.800	1.3038	1.9931	2.995
2200	**0.015**	**6.818**	**1.3056**	**1.9975**	**3.005**
3000	0.022	7.333	1.3540	2.1157	3.269
9100	0.068	7.473	1.3668	2.1454	3.338
2000	0.015	7.500	1.3693	2.1511	3.351
1300	0.010	7.692	1.3868	2.1904	3.445
2700	0.022	8.148	1.4272	2.2774	3.659
8200	0.068	8.293	1.4399	2.3033	3.724
1200	0.010	8.333	1.4434	2.3104	3.743
1800	0.015	8.333	1.4434	2.3104	3.743
7500	0.068	9.067	1.5055	2.4299	4.061
1100	0.010	9.091	1.5076	2.4336	4.072
2400	0.022	9.167	1.5138	2.4450	4.103
1600	0.015	9.375	1.5309	2.4754	4.189
1000	**0.010**	**10.000**	**1.5811**	**2.5600**	**4.437**
1500	0.015	10.000	1.5811	2.5600	4.437
2200	0.022	10.000	1.5811	2.5600	4.437
6800	0.068	10.000	1.5811	2.5600	4.437
6200	0.068	10.968	1.6559	2.6742	4.796
910	0.010	10.989	1.6575	2.6765	4.803
2000	0.022	11.000	1.6583	2.6777	4.807
1300	0.015	11.538	1.6984	2.7335	4.995
5600	0.068	12.143	1.7423	2.7909	5.196
820	0.010	12.195	1.7461	2.7956	5.213
1800	0.022	12.222	1.7480	2.7980	5.222
1200	0.015	12.500	1.7678	2.8224	5.311
750	0.010	13.333	1.8257	2.8900	5.567
5100	0.068	13.333	1.8257	2.8900	5.567
1100	0.015	13.636	1.8464	2.9127	5.657
1600	0.022	13.750	1.8540	2.9210	5.690
4700	0.068	14.468	1.9018	2.9706	5.895
1500	0.022	14.667	1.9149	2.9835	5.949
680	0.010	14.706	1.9174	2.9860	5.960
1000	**0.015**	**15.000**	**1.9365**	**3.0044**	**6.040**
4300	0.068	15.814	1.9883	3.0522	6.253
620	0.010	16.129	2.0080	3.0695	6.333
910	0.015	16.484	2.0300	3.0882	6.422
1300	0.022	16.923	2.0569	3.1104	6.529
3900	0.068	17.436	2.0878	3.1350	6.650
560	0.010	17.857	2.1129	3.1542	6.748
820	0.015	18.293	2.1385	3.1731	6.846

Active Low-Pass Filter

C1	C2	n =			Gmax
pF	µF	C2/C1	Q	x	(dB)
1200	0.022	18.333	2.1409	3.1749	6.855
510	0.010	19.608	2.2140	3.2256	7.131
750	0.015	20.000	2.2361	3.2400	7.212
1100	0.022	20.000	2.2361	3.2400	7.212
3300	0.068	20.606	2.2697	3.2612	7.335
4700	**0.100**	**21.277**	**2.3063**	**3.2833**	**7.467**
1000	0.022	22.000	2.3452	3.3058	7.606
680	0.015	22.059	2.3483	3.3075	7.617
4300	0.100	23.256	2.4112	3.3416	7.836
910	0.022	24.176	2.4584	3.3656	7.997
620	0.015	24.194	2.4593	3.3660	8.000
2700	0.068	25.185	2.5092	3.3899	8.167
3900	0.100	25.641	2.5318	3.4003	8.242
560	0.015	26.786	2.5877	3.4250	8.424
820	0.022	26.829	2.5898	3.4259	8.430
3600	0.100	27.778	2.6352	3.4447	8.576
750	0.022	29.333	2.7080	3.4731	8.804
510	0.015	29.412	2.7116	3.4745	8.815
3300	0.100	30.303	2.7524	3.4894	8.940
2200	0.068	30.909	2.7798	3.4991	9.023
4700	0.150	31.915	2.8247	3.5144	9.158
680	0.022	32.353	2.8440	3.5207	9.215
3000	0.100	33.333	2.8868	3.5344	9.340
2000	0.068	34.000	2.9155	3.5433	9.424
4300	0.150	34.884	2.9531	3.5545	9.532
620	0.022	35.484	2.9784	3.5618	9.604
2700	0.100	37.037	3.0429	3.5797	9.785
3900	0.150	38.462	3.1009	3.5948	9.944
5600	0.220	39.286	3.1339	3.6031	10.034
2400	0.100	41.667	3.2275	3.6252	10.283
3600	0.150	41.667	3.2275	3.6252	10.283
1500	0.068	45.333	3.3665	3.6548	10.640
2200	**0.100**	**45.455**	**3.3710**	**3.6557**	**10.652**
4700	0.220	46.809	3.4208	3.6655	10.776
2700	0.150	55.556	3.7268	3.7172	11.506
3900	0.220	56.410	3.7553	3.7214	11.571
1200	0.068	56.667	3.7639	3.7226	11.590
3300	0.220	66.667	4.0825	3.7636	12.284
1500	0.100	66.667	4.0825	3.7636	12.284
1000	0.068	68.000	4.1231	3.7682	12.369
2200	0.150	68.182	4.1286	3.7688	12.380
910	0.068	74.725	4.3222	3.7887	12.773
2700	0.220	81.481	4.5134	3.8060	13.144
820	0.068	82.927	4.5532	3.8094	13.219

Active Low-Pass Filter

C1 pF	C2 μF	n = C2/C1	Q	x	Gmax (dB)
750	0.068	90.667	4.7610	3.8255	13.602
2200	0.220	100.000	5.0000	3.8416	14.023
1500	0.150	100.000	5.0000	3.8416	14.023
1000	0.100	100.000	5.0000	3.8416	14.023
910	0.100	109.890	5.2414	3.8557	14.429
510	0.068	133.333	5.7735	3.8809	15.261
1500	0.220	146.667	6.0553	3.8917	15.672
1000	0.150	150.000	6.1237	3.8940	15.769
1000	0.220	220.000	7.4162	3.9276	17.423

Example 1: Want a 3-dB "bump". Use C1 = 2200pF and C2 = 0.015μF.

Example 2: Want a 6-dB "bump". Use C1 = 1000pF and C2 = 0.015μF.

Example 3: Want a 7.467-dB "bump". Use C1 = 4700pF and C2 = 0.1μF.

Example 4: Want a 10.652-dB "bump". Use C1 = 2200pF and C2 = 0.1μF.

Active Low-Pass Filter

Analysis Example: This circuit is shown in "Electronic Principles, 6th Edition" by Albert Paul Malvino. It looks like a bump filter.

Example 21-5

What are the pole frequency and Q in Fig. 21-29? What are th cutoff and 3-dB frequencies?

Solution The Q and pole frequency are:

$$Q = 0.5\sqrt{\frac{C_2}{C_1}} = 0.5\sqrt{\frac{27\text{ nF}}{390\text{ pF}}} = 4.16$$

$$f_p = \frac{1}{2\pi R\sqrt{C_1C_2}} = \frac{1}{2\pi(22\text{ k}\Omega)\sqrt{(390\text{ pF})(27\text{ nF})}} = 2.23$$

Referring to Fig. 21-26, we can read the following approximat K and A_p values:

$$K_0 = 0.99$$

$$K_c = 1.38$$

$$K_3 = 1.54$$

$$A_p = 12.5\text{ dB}$$

The cutoff or edge frequency is:

$$f_c = K_cf_p = 1.38(2.23\text{ kHz}) = 3.08\text{ kHz}$$

and the 3-dB frequency is:

$$f_{3dB} = K_3f_p = 1.54(2.23\text{ kHz}) = 3.43\text{ kHz}$$

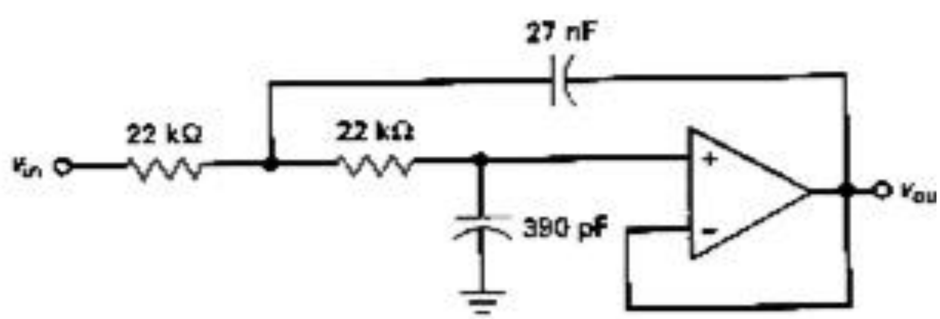


FIGURE 21-29 Unity-gain example with $Q > 0.707$.

Active Low-Pass Filter

Conventional (Malvino) Method: Calculate Q and F_p using the equations on the previous page. Then use these K values that are interpolated from a table shown partially below:

Q	K_o	K_c	K_3	$A_p\ (dB)$
0.707	-	1	1	-
2	0.935	1.322	1.485	6.3
3	0.972	1.374	1.523	9.66
4	0.984	1.391	1.537	12.1
5	0.990	1.400	1.543	14
6	0.992	1.402	1.546	15.6

Q = 4.16, so the K values are interpolated between the rows for Q = 4 & Q = 5.

Balicki Method: Use the equations on page 17:

$$x = [2 - 4\frac{C_1}{C_2}]^2, \qquad C_2 > 2C_1.$$

$$x = [2 - 4\frac{0.39nF}{27nF}]^2 = 3.7722.$$

$G_{max,dB} = -10LOG\left(1 - {}^{x}/_{4}\right) = \mathbf{12.446dB}$. Compare to $A_p\ (dB) = \mathbf{12.5dB}$.

$R = \frac{1}{2\pi F_p\sqrt{C_1 C_2}}$. Solve for F_p:

$F_p = \frac{1}{2\pi R\sqrt{C_1 C_2}} = \mathbf{2.229\ kHz}$. Compare to conventional $F_p = \mathbf{2.23\ kHz}$.

$F_c = F_p(x^{0.25}) = 2.229\ kHz\,(3.7722^{0.25}) = \mathbf{3.107\ kHz}$.

Compare to conventional $F_c = \mathbf{3.08\ kHz}$.

$F_m = \frac{F_c}{\sqrt{2}} = \mathbf{2.197\ kHz}$. Compare to conventional $F_o = K_o F_p = \mathbf{2.21\ kHz}$.

Active Low-Pass Filter

Comparison of Methods:

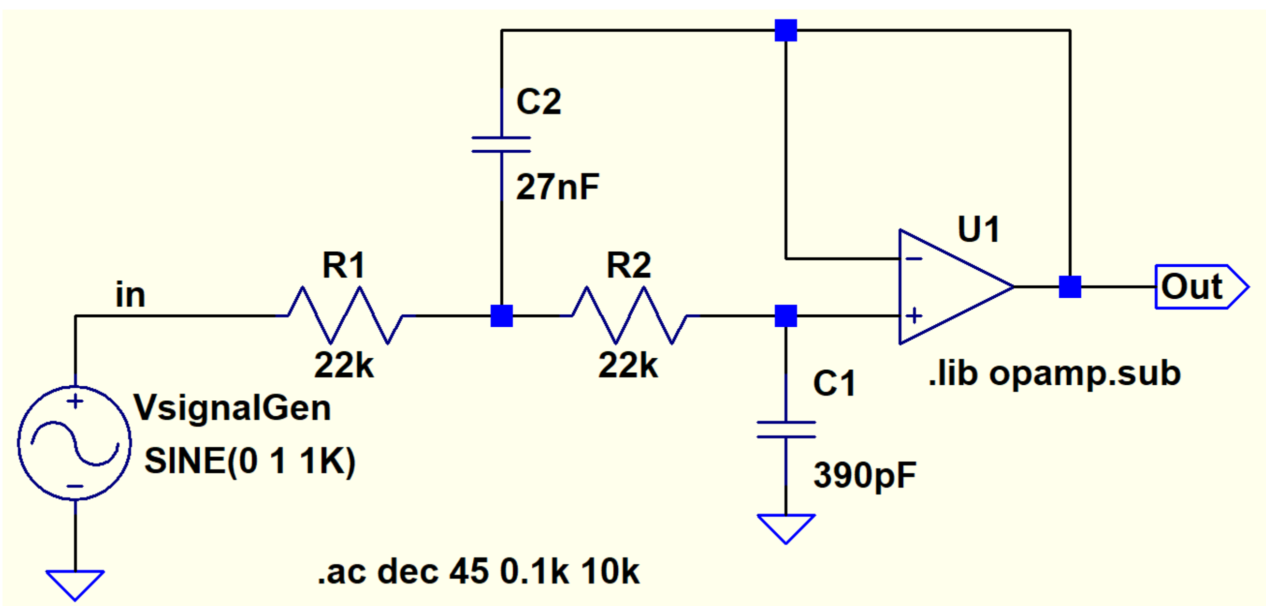
1. Both methods apply to a Sallen-Key filter with unity gain, equal resistors and a capacitor ratio > 2.
2. The "bump's" value is called $G_{max,dB}$ by Balicki, and $A_p\ (dB)$ conventionally.
3. The capacitor ratio is used to calculate a parameter called x by Balicki and a parameter called Q conventionally.
4. Balicki uses an equation, using x, to calculate $G_{max,dB}$ exactly.

   Conventionally, a table of Q and $A_p\ (dB)$ values are used. If Q's value is not an integer, interpolation is needed to get $A_p\ (dB)$.
5. Balicki uses the conventional equation for F_p . Conventionally, $F_c = K_c F_p$,

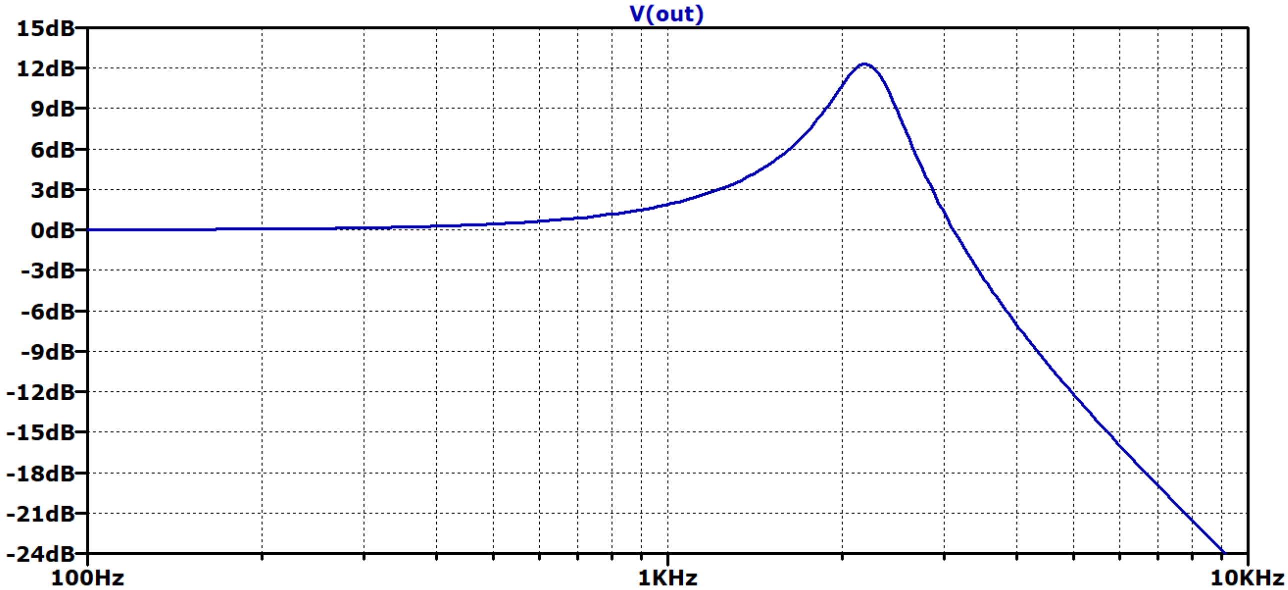
   where K_c is found in the Q table using interpolation.

   Balicki calculates F_c exactly, using the equation: $F_c = F_p(x^{0.25})$.

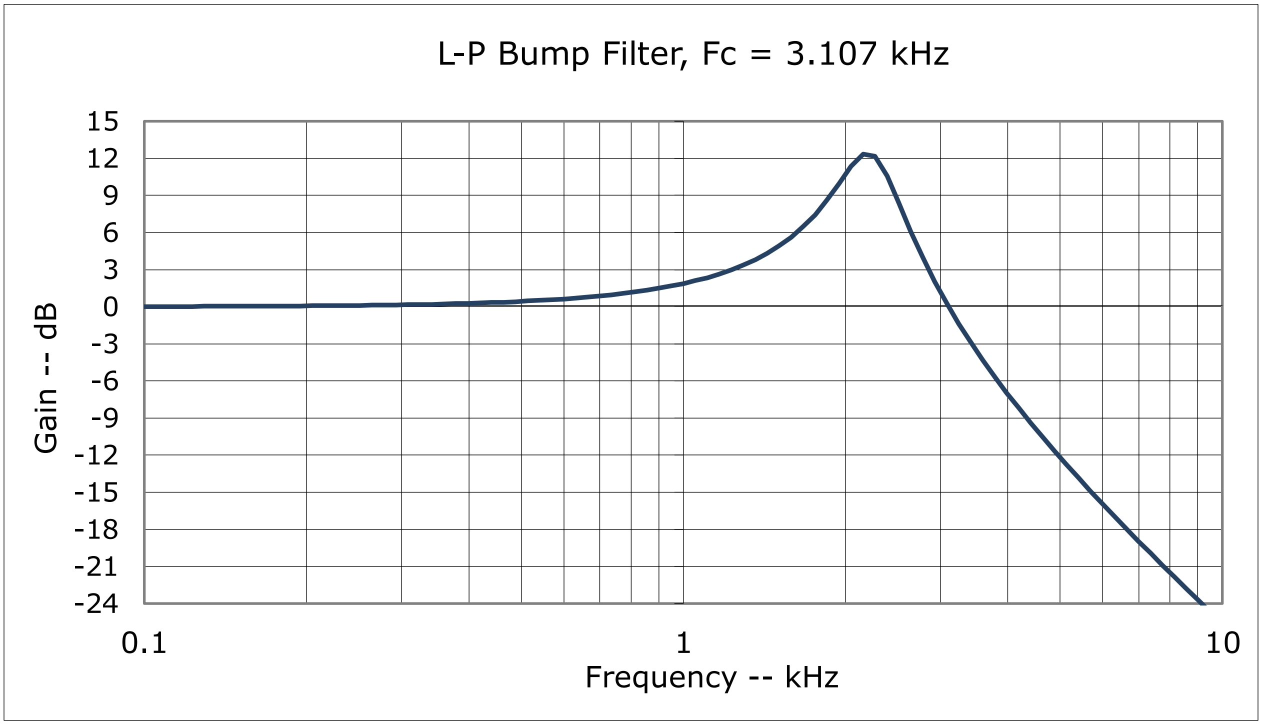
LTspice circuit:



Active Low-Pass Filter

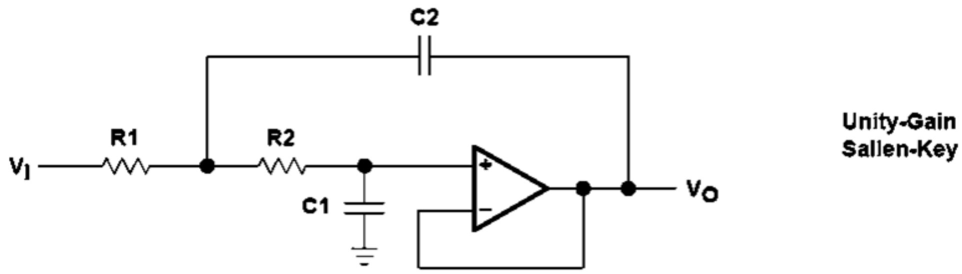


The Excel plot, below, is virtually identical to the LTspice plot above, again proving the validity of the “bump” equations.



Active Low-Pass Filter

Another Design Example: From Texas Instruments Application Report SLOA049B, "Active Low-Pass Filter Design" by Jim Karki.



R1=mR, R2=R, C1=C, C2=nC, and K=1 result in: $FSF \times fc = \frac{1}{2\pi RC\sqrt{mn}}$, and $Q = \frac{\sqrt{mn}}{m+1}$

FILTER TYPE	n	m	C1	C2	R1	R2
Butterworth	3.3	0.229	0.01 µF	0.033 µF	4.22 kΩ	18.2 kΩ
Bessel	1.5	0.42	0.01 µF	0.015 µF	7.15 kΩ	14.3 kΩ
3-dB Chebyshev	6.8	1.0	0.01 µF	0.068 µF	7.32 kΩ	7.32 kΩ

Figure 5. Sallen-Key Circuit and Component Values – fc = 1 kHz

The 3-dB 2nd Order Chebyshev filter is also a 3-dB "bump" filter.

Using the "bump" filter design equations:

Given $\boldsymbol{F_c} = \mathbf{1kHz}$ *and* $\boldsymbol{G_{max,dB}}$ ***= 3dB****.* *Select* $\boldsymbol{C_2}$ ***= 0.068μF.***

1. $F_m = F_c/\sqrt{2} = 1kHz/\sqrt{2} = 0.707\ kHz.$

2. $G_{max} = 10^{\frac{G_{max,dB}}{20}} = 10^{\frac{3}{20}} = 1.431.$

3. $1/G_{max}^2 = 1/1.431^2 = 0.501.$

4. $x = 4(1 - 1/G_{max}^2) = 4(1 - 0.501) = 1.995.$

5. $F_p = \frac{F_c}{x^{0.25}} = \frac{1kHz}{1.995^{0.25}} = 0.8414\ kHz.$

6. $C_1 = 0.25C_2\left(2 - \sqrt{x}\right) = 0.25 * 0.068\mu F\left(2 - \sqrt{1.995}\right) = 0.01\mu F$.

7. $R = \frac{1}{2\pi F_p\sqrt{C_1C_2}} = \frac{1}{2\pi * 0.814E03\sqrt{0.01E-06 * 0.068E-06}} =$ 7.259kΩ.

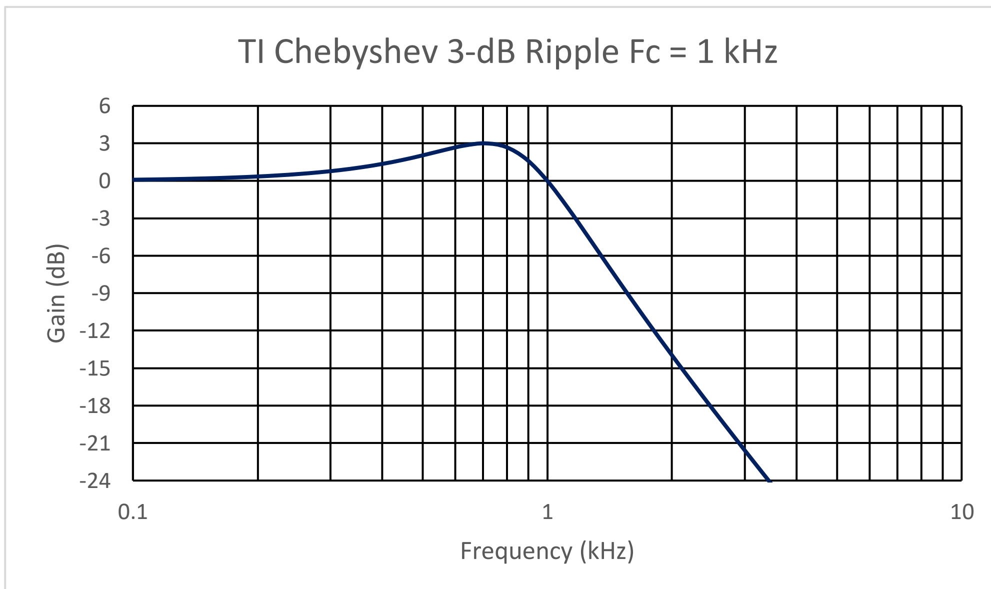
   $Use\ R = 7.32k\ \Omega$, the nearest standard value.

Active Low-Pass Filter

The Excel calculation is shown below.

Freq kHz	LP Gain dB
0.1	0.09
0.112	0.11
0.126	0.14
0.141	0.17
0.158	0.22
0.178	0.27
0.200	0.34
0.224	0.43
0.251	0.55
0.282	0.68
0.316	0.86
0.355	1.08
0.398	1.34
0.447	1.67
0.501	2.04
0.562	2.45
0.631	2.82
0.707	3.00
0.793	2.72
0.890	1.73
1.000	0.00
1.122	-2.18
1.259	-4.55
1.413	-6.96
1.585	-9.33
1.778	-11.66
1.995	-13.92
2.239	-16.14
2.512	-18.32
2.818	-20.45
3.162	-22.57
3.548	-24.66
3.981	-26.73
4.467	-28.78
5.012	-30.83
5.623	-32.86
6.310	-34.89
7.079	-36.91
7.943	-38.93
8.913	-40.95
10.000	-42.96

S/dec=	20	
Gmax (dB)	3.000	Given
Gmax	1.4125	Calc.
1/Gmax^2	0.5012	Calc.
x	1.995	Calc.
Fc (kHz)	1.000	Given
Fm (kHz)	0.707	Calc.
Fp (kHz)	0.8414	Calc.
C2 (μF)	0.068	Select
C1 (μF)	0.009987	Calc.
C1 (pF)	9987	
R (kΩ)	7.259	Calc.



Active Low-Pass Filter

Relationship between Q and x. Both Malvino and Karki use the parameter Q, whereas Balicki uses x. Karki defines m = R2/R1, and n =C2/C1.

Since R2 = R1, m = 1. Karki then states $Q = \frac{\sqrt{mn}}{m+1} = \frac{\sqrt{n}}{2}$ for $m = 1$. ----(1)

Balicki's equation: $x = [2 - 4\frac{C_1}{C_2}]^2 = (2 - \frac{4}{n})^2$----(2)

Square both sides of (1): $Q^2 = \frac{n}{4}$ or $n = 4Q^2$----(3)

Substitute (3) into (2): $x = (2 - \frac{1}{Q^2})^2$----(4), $\quad Q > \frac{1}{\sqrt{2}}$

Use (4) to solve for Q. After algebraic manipulation:

$Q = \frac{1}{\sqrt{2-\sqrt{x}}}$ ----(5), $\quad x < 4$

Example of Two Filters in Tandem Shown in Section 11-4, pp 302-304, text. The same circuit and RC values are also found in an online example.

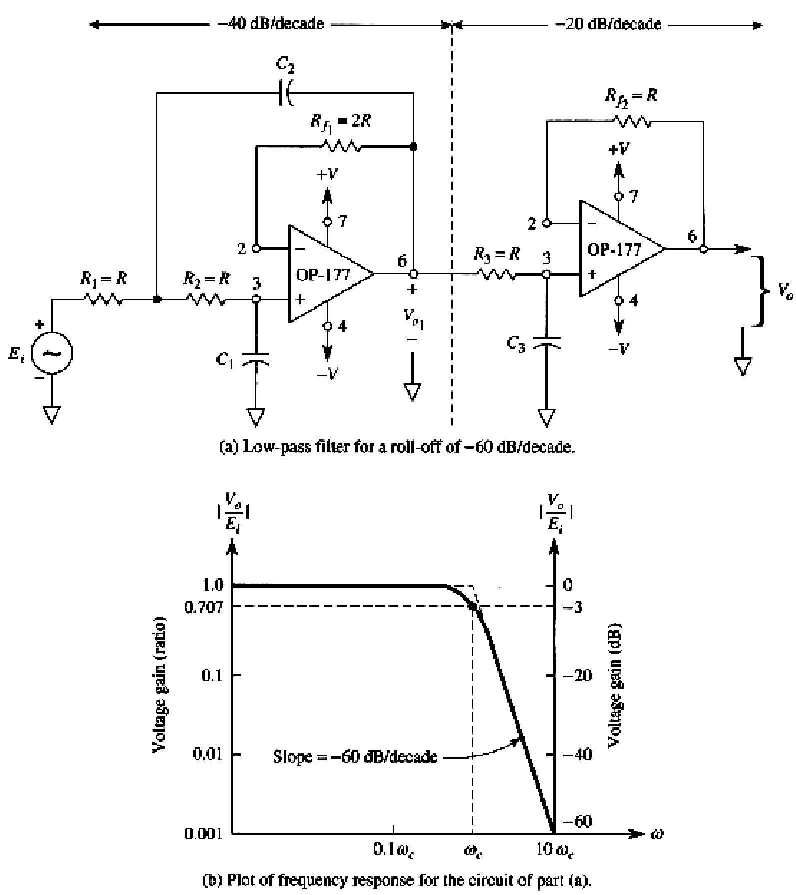


FIGURE 11-5 Low-pass filter designed for a roll-off of −60 dB/decade and corresponding frequency–response plot.

Example 11-5

For the − 60-dB/decade low-pass filter of Fig. 11-5(a), determine the values of C_1, C_2, and R for a cutoff frequency of 1 kHz. Let $C_3 = 0.01\ \mu F$.

Solution From Eq. (11-5),

$$C_1 = \tfrac{1}{2}C_3 = \tfrac{1}{2}(0.01\ \mu F) = 0.005\ \mu F$$

and

$$C_2 = 2C_3 = 2(0.01\ \mu F) = 0.02\ \mu F$$

From Eq. (11-6),

$$R = \frac{1}{(6.28)(1 \times 10^3)(0.01 \times 10^{-6})} = 15{,}915\ \Omega$$

Both authors claim that this is a 3rd order Butterworth (flat) filter, yet the 1st stage looks like a "bump" filter. For the 1st stage, n = C2/C1 = 0.02µF/0.005 µF = 4.

$$x = (2 - \frac{4}{n})^2 = (2 - \frac{4}{4})^2 = (2-1)^2 = 1.$$

$$G_{max,dB} = -10LOG\left(1 - {}^{x}/_{4}\right) = 1.249\text{dB}.$$

$$F_c = F_p(x^{0.25}) = F_p(1^{0.25}) = F_p.$$

So the 1st stage has a "bump" of 1.249dB, and is followed by a 2nd stage that is a simple 1st order (1 pole) filter. Both stages have the same F_c value.

Derivation of the transfer function:

1st stage:

$$\frac{V_{o1}}{V_{in}} = \frac{1}{\sqrt{1 - x\ (F/F_c)^2 + x\ (F/F_c)^4}}$$

2nd stage:

$$\frac{V_o}{V_{o1}} = \frac{1}{\sqrt{1 + (F/F_c)^2}}$$

Active Low-Pass Filter

Total filter = (1st stage) * (2nd stage): $\frac{V_o}{V_{in}} = \frac{V_{o1}}{V_{in}} * \frac{V_o}{V_{o1}}$

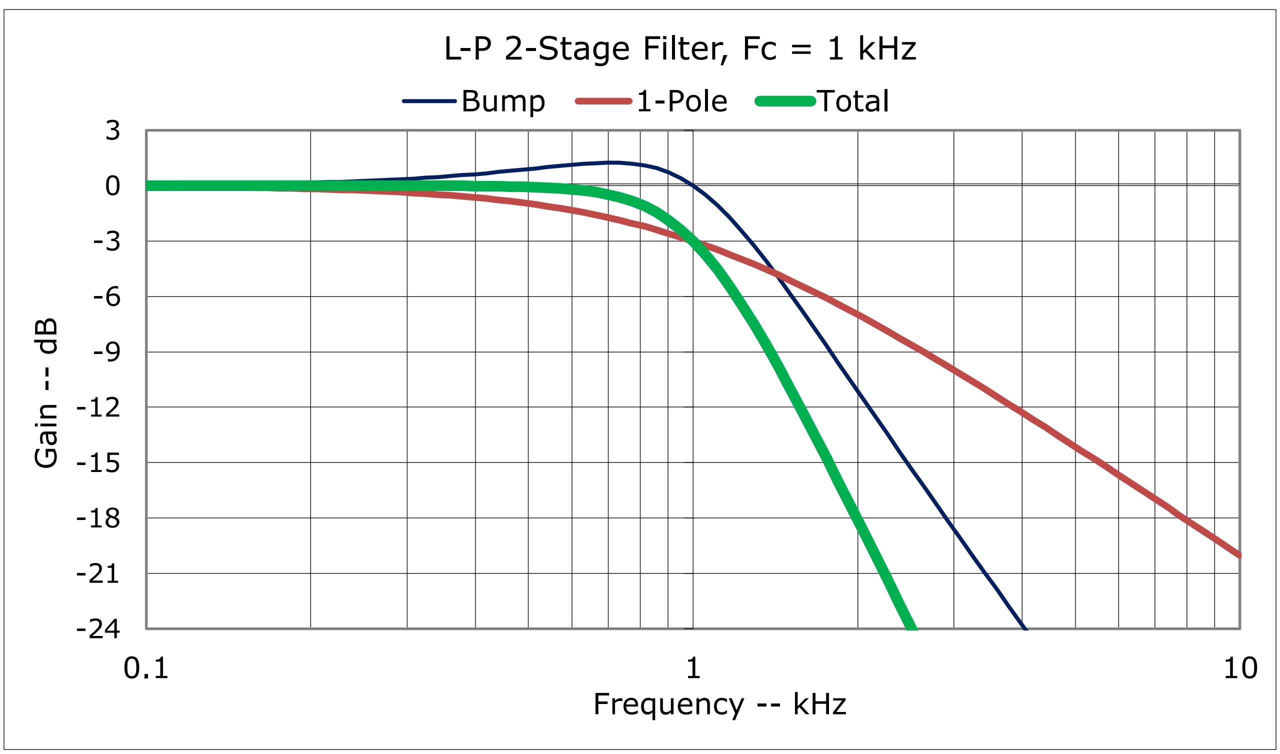
$$\frac{V_o}{V_{in}} = \frac{1}{\sqrt{[1 - x\,(F/F_c)^2 + x\,(F/F_c)^4] * [1 + (F/F_c)^2]}}$$

$$\frac{V_o}{V_{in}} = \frac{1}{\sqrt{1 - x\,(F/F_c)^2 + x(F/F_c)^4 + (F/F_c)^2 - x\,(F/F_c)^4 + x\,(F/F_c)^6}}$$

$$\frac{V_o}{V_{in}} = \frac{1}{\sqrt{1 + (1 - x)\,(F/F_c)^2 + x\,(F/F_c)^6}}$$

$$\frac{V_o}{V_{in}} = \frac{1}{\sqrt{1 + (F/F_c)^6}}$$ for $x = 1.$

The result is a 3rd order Butterworth (flat) filter as claimed. There is no "bump" in the total filter's Bode plot:



Capacitor values: Use two 0.01μF in parallel for 0.02μF. Use two 0.01μF in series for 0.005μF. So the circuit uses a total of five 0.01μF capacitors.

Attenuation of 60dB/decade is the same as 18dB/octave. For the total gain above:

- Gain = -3dB at $F_c = 1kHz$.
- Gain = -18dB at $2\ F_c = 2kHz$.

In summary, neither the online author, nor the text authors, acknowledge a "bump" filter or its equation using the parameter x. To get a 3rd order Butterworth filter:

- The 2nd order filter must be a "bump" filter.
- $x = 1$ for the "bump" filter.

Example of Three Filters in Tandem

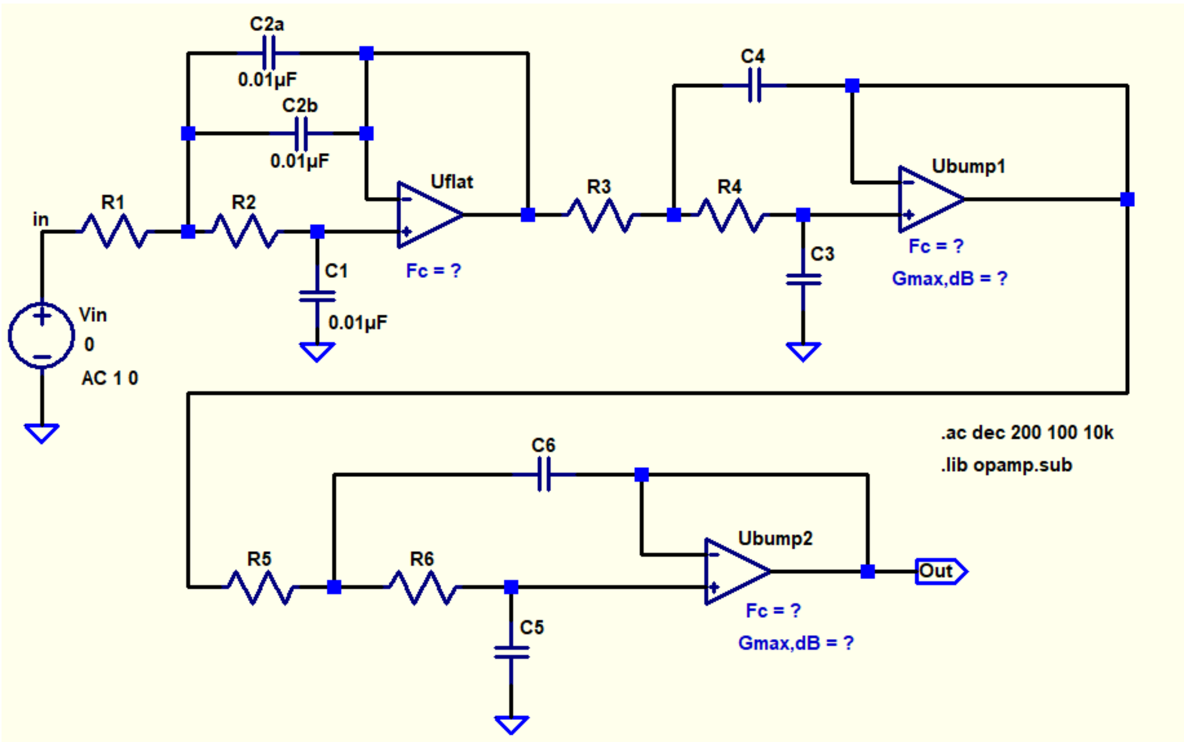
Combinations of "bump" and flat filters can be connected in tandem to meet specifications. Example of lenient specifications:

1. 3dB maximum gain in the passband.
2. Gain at 3 kHz must be no lower than -3dB.
3. Attenuation of at least 14dB at 4 kHz.

Such a filter might be used to filter the signal at the input to an A/D converter that samples an analog signal at 8 kHz. At least 14dB of attenuation is required at half of the sampling rate.

Active Low-Pass Filter

Example 1: A combination of one flat filter and two “bump” filters does the job.



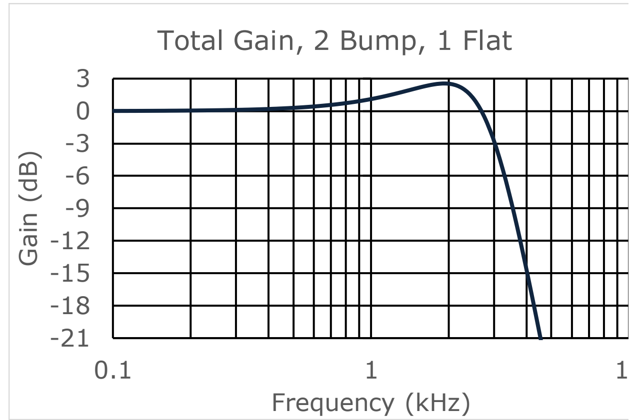
The design job is to select the three F_c values and the two $G_{max,dB}$ values so that the filter meets the above specifications. To start, construct a table in an Excel spreadsheet that uses the three F_c values and the two $G_{max,dB}$ values to calculate the filter parameters. Select $G_{max,dB}$ values from the capacitor table, Pages 19-21.

	Bump	**Bump**	**Flat**
	G1	**G2**	**Gflat**
Gmax (dB)	1.634	1.634	
Gmax	1.207	1.207	
1/Gmax^2	0.686	0.686	
x	1.254	1.254	
Fc (kHz)	2.990	2.990	3.100
Fm (kHz)	2.114	2.114	
Fp (kHz)	2.825	2.825	
C2 (µF)	0.01	0.01	0.02
C1 (µF)	0.002200	0.002200	0.01
C1 (pF)	2200	2200	
R (kΩ)	12.009	12.009	3.630

Active Low-Pass Filter

Next, add 4 columns to the spreadsheet to calculate the gain of each of the three filters at 45 frequencies per decade. Add a 5th column to calculate the total gain which is the sum (in dB) of the three individual gains.

Freq	Bump1	Bump2	Flat	Total
kHz	**dB**	**dB**	**dB**	**dB**
0.100	0.01	0.01	0.00	0.01
0.105	0.01	0.01	0.00	0.01
0.111	0.01	0.01	0.00	0.01
0.117	0.01	0.01	0.00	0.02
0.123	0.01	0.01	0.00	0.02
0.129	0.01	0.01	0.00	0.02
0.136	0.01	0.01	0.00	0.02
0.143	0.01	0.01	0.00	0.02
0.151	0.01	0.01	0.00	0.03
0.158	0.02	0.02	0.00	0.03
0.167	0.02	0.02	0.00	0.03
0.176	0.02	0.02	0.00	0.04
0.185	0.02	0.02	0.00	0.04
0.194	0.02	0.02	0.00	0.05
0.205	0.03	0.03	0.00	0.05
0.215	0.03	0.03	0.00	0.06
0.227	0.03	0.03	0.00	0.06
0.239	0.03	0.03	0.00	0.07
0.251	0.04	0.04	0.00	0.08
0.264	0.04	0.04	0.00	0.08
0.278	0.05	0.05	0.00	0.09
0.293	0.05	0.05	0.00	0.10
0.308	0.06	0.06	0.00	0.11
0.324	0.06	0.06	0.00	0.13
0.341	0.07	0.07	0.00	0.14
0.359	0.08	0.08	0.00	0.16
0.378	0.09	0.09	0.00	0.17
0.398	0.10	0.10	0.00	0.19
0.419	0.11	0.11	0.00	0.21
0.441	0.12	0.12	0.00	0.23
0.464	0.13	0.13	0.00	0.26
0.489	0.14	0.14	0.00	0.29
0.514	0.16	0.16	0.00	0.32
0.541	0.18	0.18	0.00	0.35
0.570	0.19	0.19	0.00	0.38
0.599	0.22	0.22	-0.01	0.42
0.631	0.24	0.24	-0.01	0.47
0.664	0.26	0.26	-0.01	0.52
0.699	0.29	0.29	-0.01	0.57
0.736	0.32	0.32	-0.01	0.63
0.774	0.35	0.35	-0.02	0.69
0.815	0.39	0.39	-0.02	0.76



Excel plot using the column data shown.

Active Low-Pass Filter

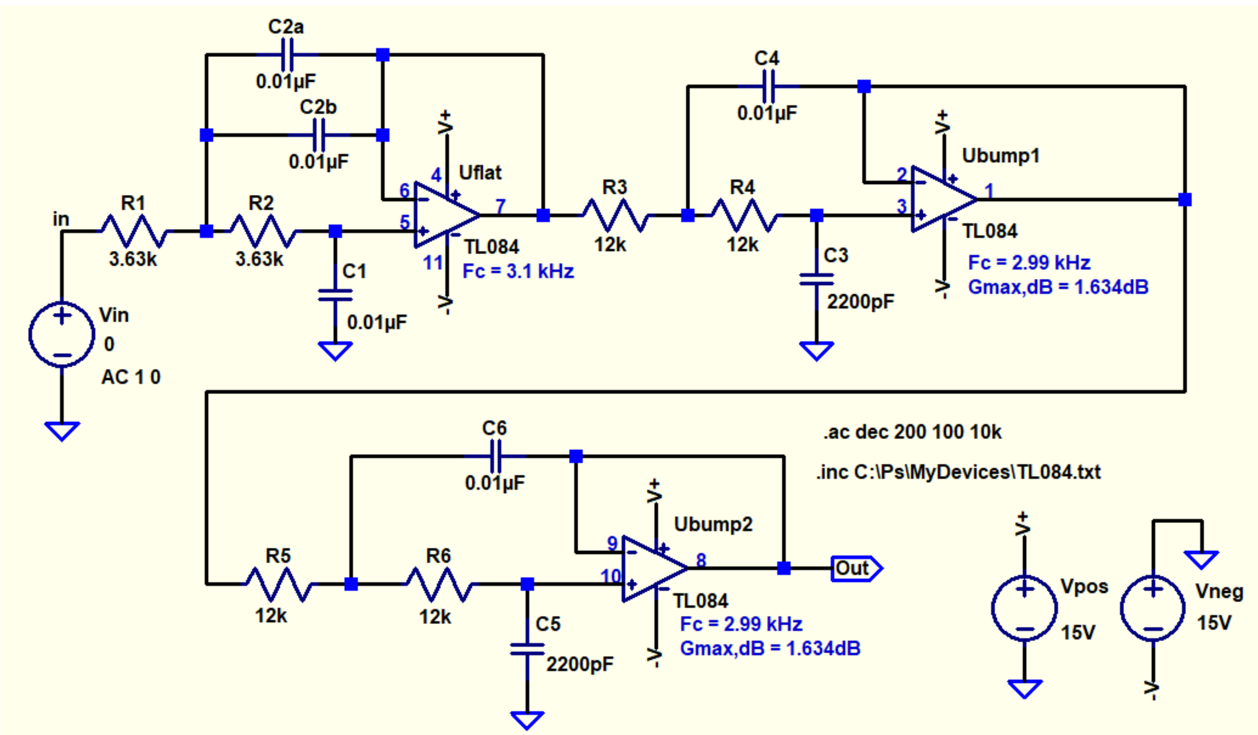
Freq	Bump1	Bump2	Flat	Total
kHz	**dB**	**dB**	**dB**	**dB**
0.858	0.43	0.43	-0.03	0.84
0.903	0.48	0.48	-0.03	0.92
0.950	0.52	0.52	-0.04	1.01
1.000	0.58	0.58	-0.05	1.11
1.053	0.64	0.64	-0.06	1.21
1.108	0.70	0.70	-0.07	1.33
1.166	0.77	0.77	-0.09	1.45
1.227	0.84	0.84	-0.11	1.57
1.292	0.92	0.92	-0.13	1.71
1.359	1.00	1.00	-0.16	1.84
1.431	1.09	1.09	-0.19	1.98
1.506	1.18	1.18	-0.24	2.12
1.585	1.27	1.27	-0.29	2.25
1.668	1.36	1.36	-0.35	2.37
1.756	1.45	1.45	-0.43	2.47
1.848	1.52	1.52	-0.52	2.53
1.945	1.59	1.59	-0.63	**2.55**
2.047	1.63	1.63	-0.76	2.50
2.154	1.63	1.63	-0.91	2.35
2.268	1.59	1.59	-1.09	2.09
2.387	1.49	1.49	-1.31	1.67
2.512	1.31	1.31	-1.56	1.06
2.644	1.05	1.05	-1.84	0.25
2.783	0.68	0.68	-2.17	-0.81
2.929	0.22	0.22	-2.54	-2.11
3.000	-0.04	-0.04	-2.73	**-2.81**
3.082	-0.35	-0.35	-2.96	-3.66
3.244	-1.01	-1.01	-3.42	-5.44
3.415	-1.75	-1.75	-3.93	-7.44
3.594	-2.57	-2.57	-4.48	-9.61
3.782	-3.43	-3.43	-5.07	-11.94
3.981	-4.34	-4.34	-5.71	-14.39
4.000	-4.43	-4.43	-5.77	**-14.62**
4.190	-5.28	-5.28	-6.37	-16.94
4.410	-6.24	-6.24	-7.07	-19.55
4.642	-7.21	-7.21	-7.80	-22.22
4.885	-8.19	-8.19	-8.55	-24.93
5.142	-9.17	-9.17	-9.33	-27.67
5.412	-10.15	-10.15	-10.12	-30.42
5.696	-11.13	-11.13	-10.93	-33.19
5.995	-12.10	-12.10	-11.76	-35.96
6.310	-13.07	-13.07	-12.59	-38.74
6.641	-14.04	-14.04	-13.44	-41.51
6.989	-15.00	-15.00	-14.29	-44.28
7.356	-15.95	-15.95	-15.15	-47.05
7.743	-16.90	-16.90	-16.01	-49.81

Active Low-Pass Filter

The total gain at the three critical values (Max, 3 kHz, & 4 kHz) are shown in bold in the above table. For convenience, add the following to the spreadsheet:

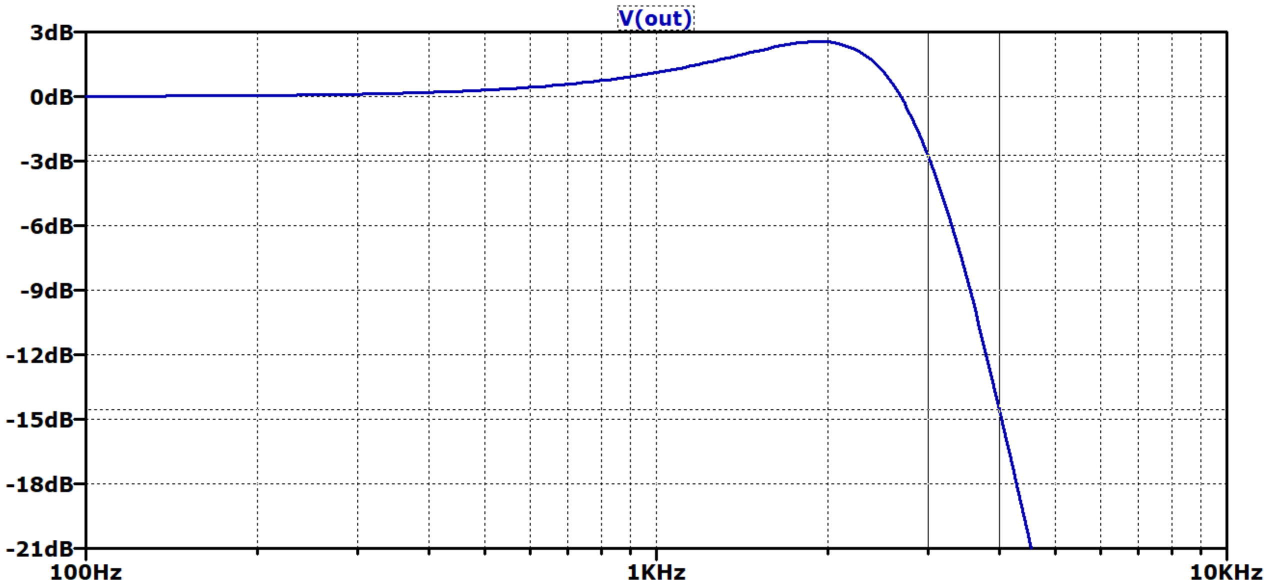
	Bump G1	Bump G2	Flat Gflat		Freq kHz	Total dB	Req'm't	Meet Req'm't
Gmax (dB)	1.634	1.634			**Max**	**2.55**	**3.00**	**YES**
Gmax	1.207	1.207			**3.000**	**-2.81**	**-3.00**	**YES**
1/Gmax^2	0.686	0.686			**4.000**	**-14.62**	**-14.00**	**YES**
x	1.254	1.254						
Fc (kHz)	2.990	2.990	3.100					
Fm (kHz)	2.114	2.114						
Fp (kHz)	2.825	2.825						
C2 (µF)	0.01	0.01	0.02					
C1 (µF)	0.002200	0.002200	0.01					
C1 (pF)	2200	2200						
R (kΩ)	12.009	12.009	3.630					

While a ***trial-and-error process***, use of a spreadsheet quickly finds a solution that meets all three requirements. Here, the two $G_{max,dB}$ values were chosen from the capacitor table so that C1 and C2 had standard values. Then various choices were made for the three F_c values until the design met all three requirements.



LTspice circuit above, and resulting plot on the next page.

Active Low-Pass Filter



Using cursors,

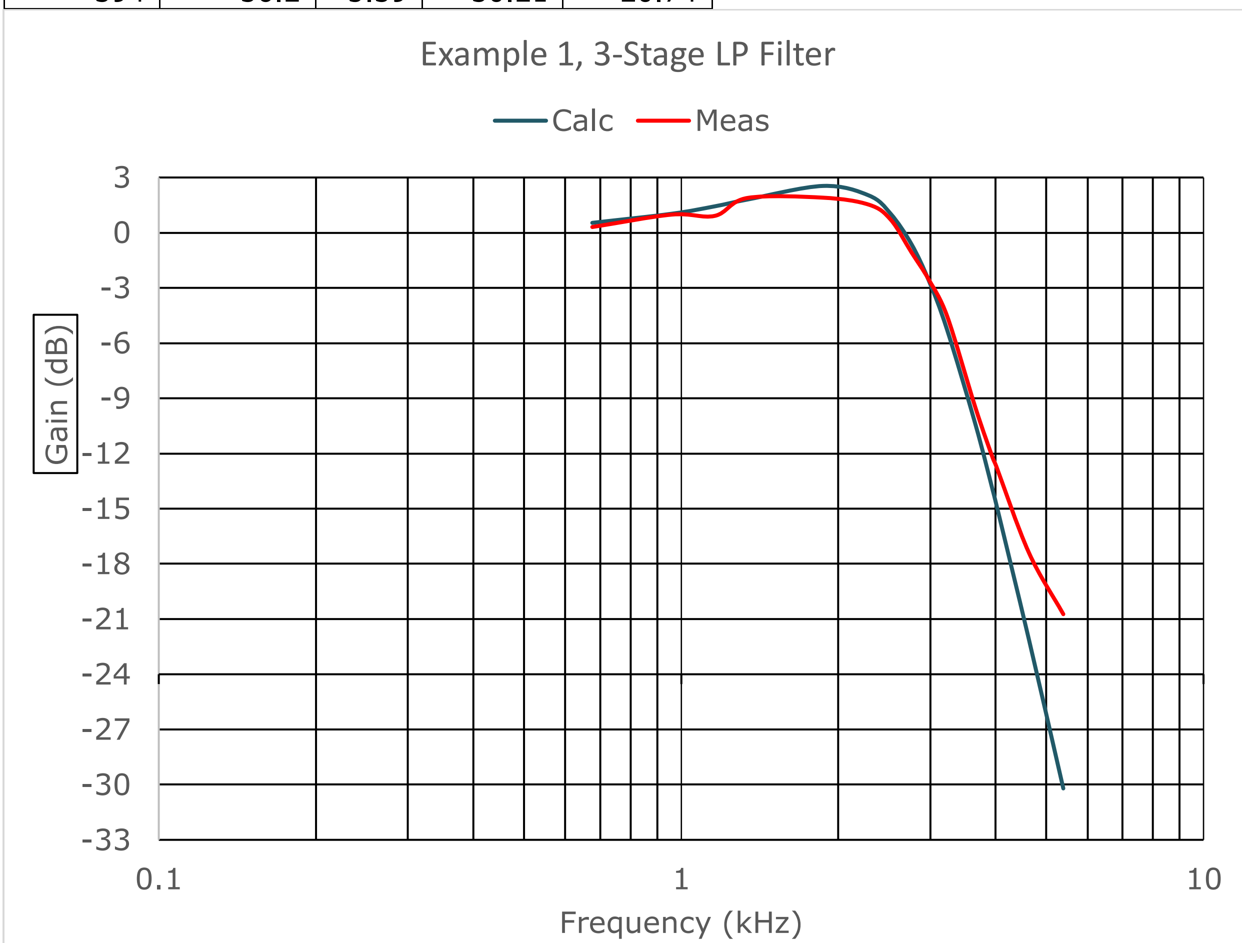
At 3 kHz, Gain =-2.74dB.

At 4 kHz, Gain =-14.54dB.

Active Low-Pass Filter

Measured Results, Example 1:

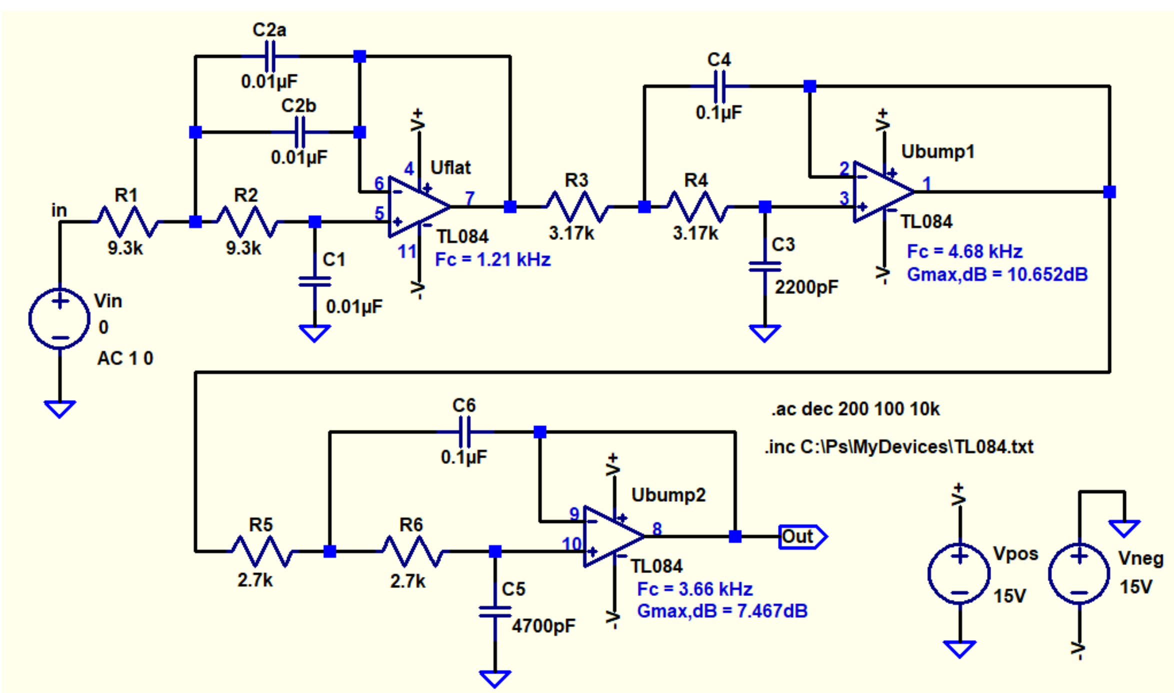
Vin	Vout	**kHz**	**dB**	**dB**
mV, rms	mV, rms	**Freq**	**Calc**	**Meas**
415	430	0.676	0.54	0.31
392	439	0.959	1.03	0.98
405	450	1.16	1.43	0.92
391	486	1.34	1.80	1.89
404	503	1.87	2.54	1.90
403	480	2.29	2.02	1.52
393	426	2.52	1.02	0.70
415	367	2.76	-0.62	-1.07
404	301	2.98	-2.61	-2.56
391	232	3.23	-5.28	-4.53
407	136	3.66	-10.43	-9.52
402	98.1	3.95	-14.01	-12.25
399	92.6	4.01	-14.75	-12.69
404	54.6	4.63	-22.09	-17.38
394	36.2	5.39	-30.21	-20.74



Active Low-Pass Filter

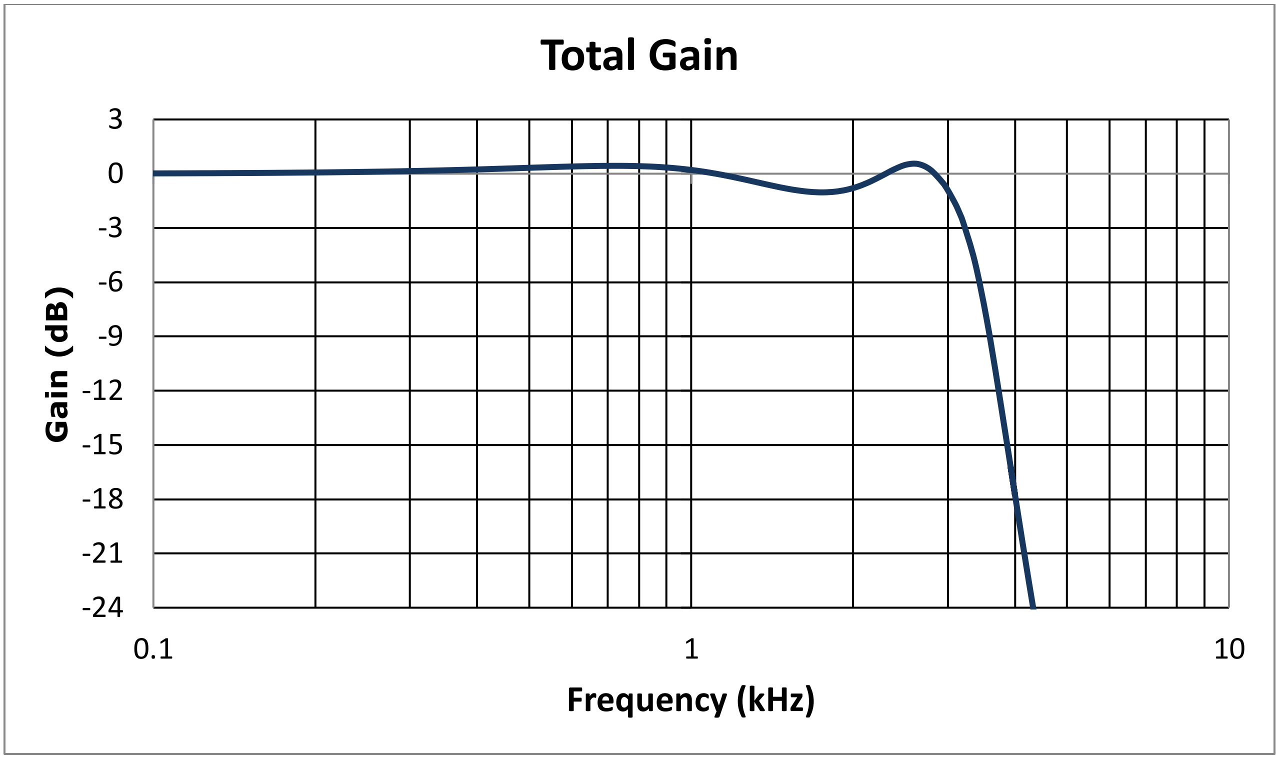
Example 2: The requirements used in the previous example might be, probably are, too lenient. There is an allowable gain variation of 6dB between dc and 3 kHz. In this new example, the maximum allowable gain variation (passband ripple) is 3dB. For example, if the maximum gain turns out to be +1dB, then the gain at 3 kHz is must be > – 2dB. That is, gain at 3 kHz must be > (MAX – 3dB).
Two "bumps" and one flat are again used to meet this new requirement. Compared to the previous example, much larger $G_{max,dB}$ values are used. To compensate, the flat filter's F_c is a relatively low value. See table below:

	Bump	Bump	Flat		Freq	Total		Meet	
	G1	G2	Gflat		kHz	dB	Req'm't	Req'm't	Margin
Gmax (dB)	10.652	7.467			Max	0.53	3.00	YES	2.47
Gmax	3.409	2.362			3.000	-0.91	-2.47	YES	1.56
1/Gmax^2	0.086	0.179			4.000	-17.74	-14.00	YES	3.74
x	3.656	3.283			Min < 3 kHz	-1.03	-2.47	YES	1.44
Fc (kHz)	4.680	3.660	1.210		Max - Min	1.56	3.00	YES	1.44
Fm (kHz)	3.309	2.588							
Fp (kHz)	3.385	2.719							
C2 (µF)	0.1	0.1	0.02						
C1 (µF)	0.002200	0.004701	0.01						
C1 (pF)	2200	4701							
R (kΩ)	3.170	2.700	9.301						

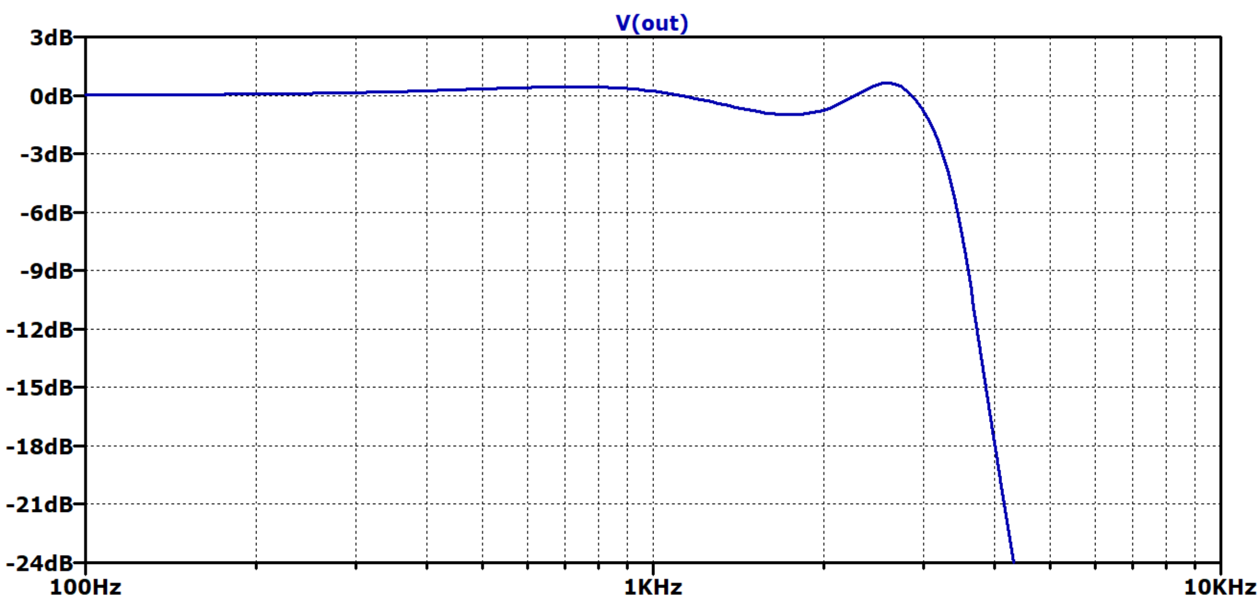


LTspice circuit

Active Low-Pass Filter

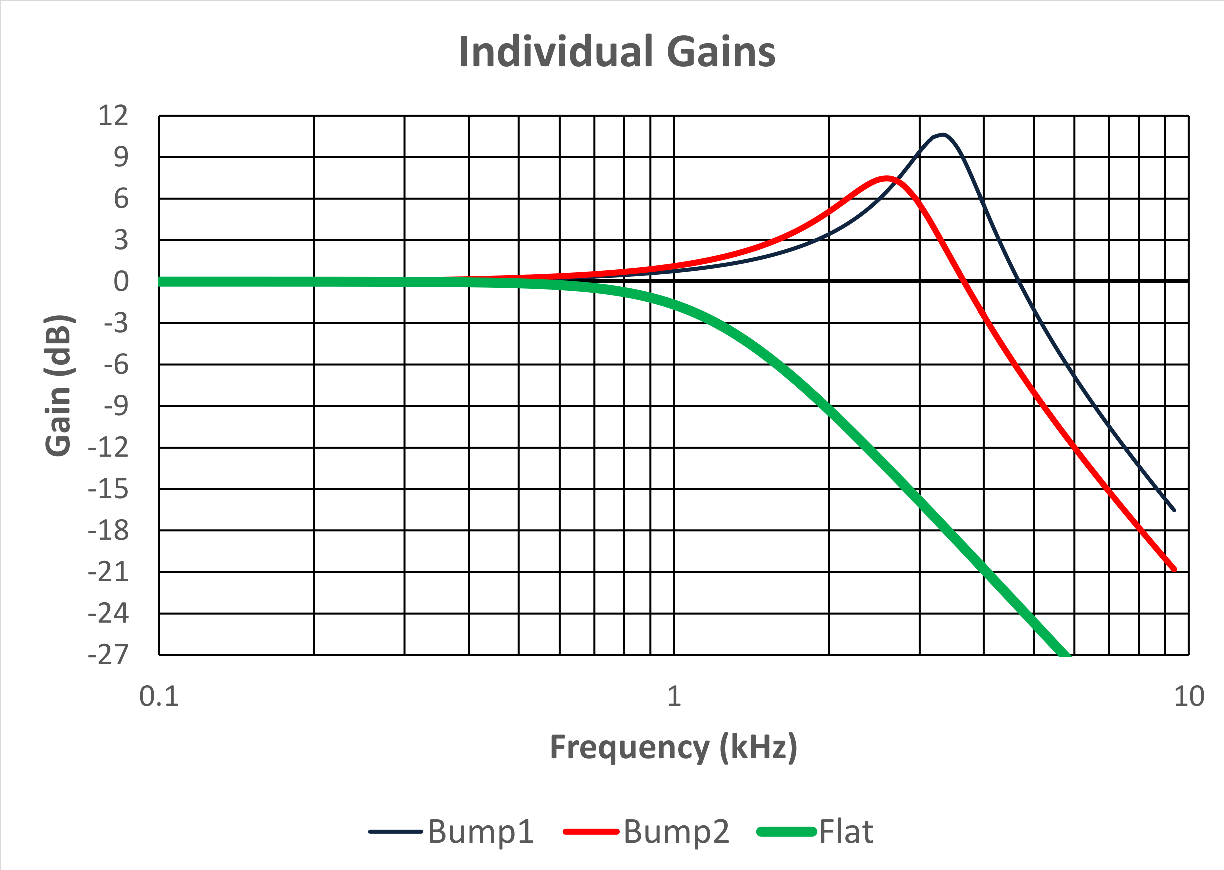


Excel plot above, LTspice plot below.



The two graphs above are virtually identical, again proving that the "bump" equations are correct.

Active Low-Pass Filter

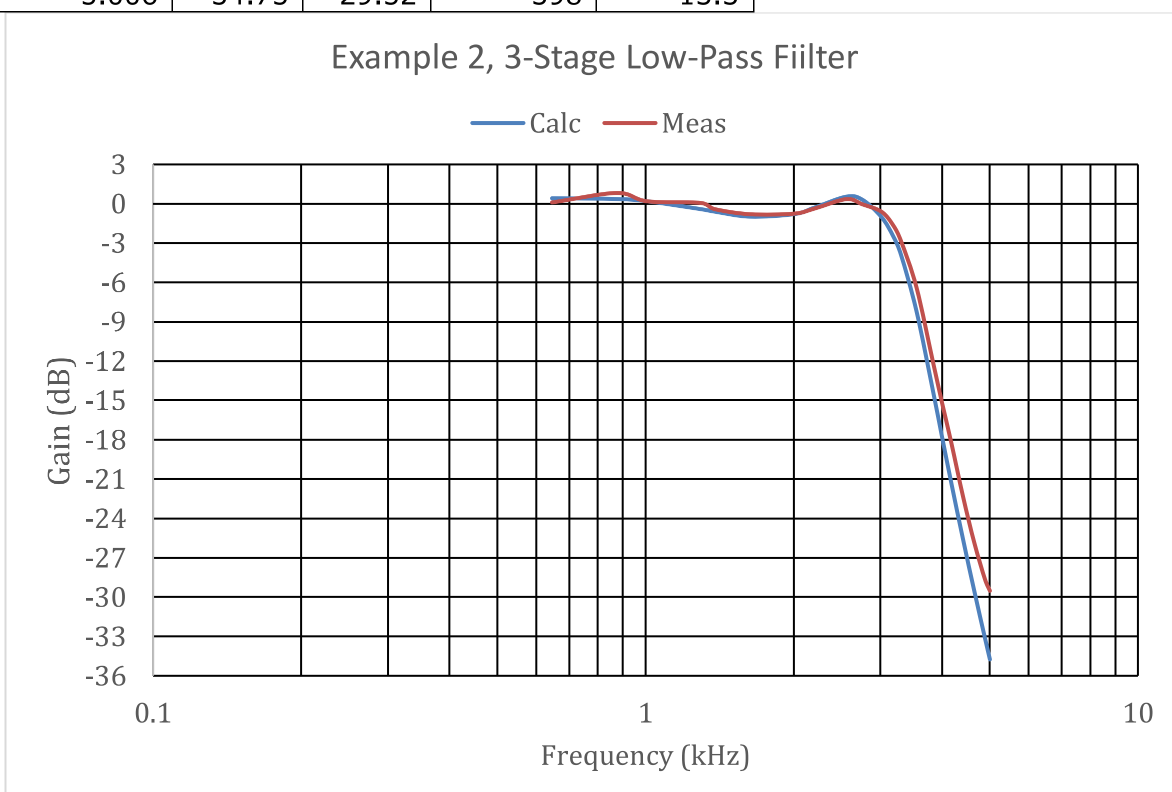


Excel plot. Note that the largest $G_{max,dB}$ corresponds to the largest F_c.

Active Low-Pass Filter

Measured Results, Example 2:

Freq kHz	dB Calc	dB Meas	Vin mV, rms	Vout mV, rms
0.646	0.42	0.10	416	421
0.875	**0.37**	**0.83**	390	429
1.007	0.20	0.19	403	412
1.296	-0.41	0.07	392	395
1.381	-0.59	-0.40	399	381
1.631	-0.98	-0.80	400	365
2.006	**-0.78**	**-0.75**	400	367
2.203	-0.29	-0.37	388	372
2.557	0.54	0.36	406	423
2.759	0.30	-0.04	412	410
3.016	**-1.02**	**-0.61**	400	373
3.187	-2.50	-1.66	391	323
3.305	-3.93	-2.87	391	281
3.552	-8.21	-6.39	403	193
3.838	-14.32	-12.11	407	101
4.003	**-17.81**	**-15.20**	400	69.5
4.180	-21.31	-18.24	393	48.1
4.418	-25.65	-22.38	396	30.1
4.653	-29.53	-25.85	404	20.6
4.891	-33.13	-28.64	403	14.9
5.006	-34.75	-29.52	398	13.3



Active Low-Pass Filter

Discussion and Summary:

Chapter 1 starts with the basic Sallen-Key equation in the s domain. Conventional algebra converts the s-domain equation to a frequency-domain equation. Then a frequency-domain equation is presented for the 2nd order flat (Butterworth) filter. Equations are then derived for the R & C values by equating the coefficients of frequency for the two previous equations. It is found that the capacitor ratio must be two for a flat Sallen-Key filter.

Chapter 2 presents a unique frequency-domain equation for a "bump" Sallen-Key filter. Re-using the basic Sallen-Key equation from Chapter 1, and this new unique equation, R & C values are again derived by equating the coefficients of frequency. It is found that the capacitor ratio must be greater than two for the "bump" filter. In fact, the capacitor ratio controls the value of the "bump". The larger the ratio, the larger the "bump". For a practical filter, the "bump" should probably limited to 3dB, if only one filter stage is used.

However, "bump" values much larger than 3dB can be used effectively in multi-stage filters. Thus combinations of "bump" and flat filters can be used to meet design specifications. While a trial-and-error process, use of a spreadsheet quickly finds a solution that meets requirements.